Skeptical Argument and Moral Vagueness in Moral Epistemology: Ethical Intuitionism Treatment by Analogy with Mathematical Knowledge

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Abstract

Skeptical argument has been bothering epistemologists for a long time. Yet, if one tries to grasp its theoretical challenge, she is likely to encounter a set of somewhat bizarre thought experiments, e.g. “brain-in-a-vat”, “epistemological evil demon”, and the like (Pritchard, 2009). The pressure of skeptical argument remains mostly abstract for lay people. Unfortunately, in the area of Moral epistemology, the situation seems to be different. A moral skeptic has at her disposal a very reasonable hypothesis asserting that there is no moral knowledge: the evolutionary explanation of the genealogy of our moral beliefs (Lutz, 2015; Kappel, 2002). Thus, all our moral beliefs are mere rationalizations of our best surviving strategies. This hypothesis is elegant, simple, and explanatory. Thus, one who is uncomfortable adopting moral skepticism should propose at least a similarly reasonable non-skeptical explanation. One candidate for such an explanation is an analogy between mathematical knowledge and moral knowledge (Korchevoi, 2023a). At first glance, this analogy may seem to steer us too far from the real ground of common sense; it is too abstract. Yet, it is easy to see that this analogy stubbornly occurs in ethical thinking (Ross, 1930; Audi, 2004, 2008; Lutz, 2015; Clarke-Doane, 2019). This dissertation aims to develop and refine arguments refuting the hold of Skeptical argument in Moral epistemology using explanatory power of the concept of “self-evident” propositions. The main thread of argumentation will borrow the epistemic ground from the several approaches to the philosophy of mathematics, in particular Platonism, Logicism, and mathematical Intuitionism. Thus, it is unlikely that a proponent of strong fallibilism in mathematics will be convinced by our arguments. Yet, we hope to show that if one believes in a solid epistemological ground of several mathematical propositions, she should believe in the existence of similar epistemological ground for some propositions in Moral epistemology.

Key words: skeptical hypothesis, moral epistemology, ethical-mathematical analogy, self-evident proposition.

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1. Introduction and the aim of the study. Permeation of Moral Skepticism into moral judgements

Skeptical argument has been bothering epistemologists for a long time. Yet, if one tries to grasp its theoretical challenge, she is likely to encounter a set of somewhat bizarre thought experiments, e.g. “brain-in-a-vat”, “epistemological evil demon”, and the like (Pritchard, 2009). The pressure of skeptical argument remains mostly abstract for lay people. Unfortunately, in the area of Moral epistemology, the situation seems to be different. A moral skeptic has at her disposal a very reasonable hypothesis asserting that there is no moral knowledge, at least there is no non-natural moral knowledge: the evolutionary explanation of the genealogy of our moral beliefs (Lutz, 2015; Kappel, 2002). Thus, all our moral beliefs are mere rationalizations of our best surviving strategies. This hypothesis is elegant, simple, and explanatory. Yet, surrendering to the attack of Skeptical hypothesis is troublesome for many in Moral epistemology because, paraphrasing Dostoevsky (1880), if there isn’t a non-natural moral law, then everything is permissible. However, arguing in favor of the existence of non-natural moral knowledge is a task even more difficult than trying to convince a skeptic that I am not a “brain-in-a-vat”.

Thus, one who is uncomfortable adopting moral skepticism should propose at least a similarly reasonable non-skeptical explanation. One candidate for such an explanation is an analogy between mathematical knowledge and moral knowledge (Korchevoi, 2023a). At first glance, this analogy may seem to steer us too far from the real ground of common sense; it is too abstract. Yet, it is easy to see that this analogy stubbornly occurs in ethical thinking (Ross, 1930; Audi, 2004, 2008; Lutz, 2015; Clarke-Doane, 2019). This dissertation aims to develop and refine arguments refuting the hold of Skeptical argument in Moral epistemology using explanatory power of the concept of “self-evident” propositions. The main thread of argumentation will borrow the epistemic ground from the several approaches to the philosophy of mathematics, in particular Platonism, Logicism, and mathematical Intuitionism. Thus, it is unlikely that a proponent of strong fallibilism in mathematics will be convinced by our arguments. Yet, we hope to show that if one believes in a solid epistemological ground of several mathematical propositions, she should believe in the existence of similar epistemological ground for some propositions in Moral epistemology.

The main aim of this study is to develop the idea that there is a non-inferential knowledge of basic so-called "self-evident" moral propositions. Thus, we will consider the domain of epistemology of moral, or ethical, Intuitionism. There are several possible lines of argumentation for defending Intuitionism, e.g. moral knowledge which is analogous to perceptual knowledge (Lutz, 2015; Kappel, 2002). However, we will develop further ethical-mathematical analogy with particular attention to mathematics. When moral epistemologists talk about this matter, they cite works of moral epistemologists such as Audi (2008) or Huemer (2005), see, for example, Lutz (2015). In other cases, they note ideas of Ross (1930), e.g. 'the self-evidence of moral truths is analogous to the self-evidence of mathematical axioms' (Bedke, 2010: 1070). One can be concerned why those argumentations seem to lack coherence in references to certain mathematical concepts. The above argumentations seem to have made several implicit assumptions about philosophy of mathematics. Consequently, the argumentations employ mathematical propositions without giving a rationale why a particular mathematical proposition was chosen to support or refute an argument. Thus, our secondary aim is to present a coherent survey related to current debate in the philosophy of mathematics for one to draw ethical-mathematical analogy.

We can find a more or less extensive analysis of the above mathematical analogy in Lutz (2015). In his arguments, such models of epistemology of mathematical knowledge as mathematical Intuitionism and Nominalism are not deemed to fit the purpose of the analogy. Thus, Lutz (2015) moves quickly to the model of conceptual mathematical knowledge. We agree with his distrust of Nominalism. However, the question "is mathematics discovered or invented?" is not a platitude (Ernest, 1999). The critique of the absolutist view of mathematics sounds reasonable until we trace such debates to the point where mathematics, even pure theoretical kind, becomes a mere social construct. For the sake of clarity of our consideration, we must explicitly state our premises that at least pure theoretical mathematics is infallible and presupposes understanding and/or knowledge of several mathematical concepts and relations among them which can be expressed as true propositions. We will provide extensive consideration for why those premises have solid epistemological ground. Yet, while pure mathematics is infallible, mathematicians, as well as some ramifications of applied mathematics, are fallible - some of them rather easily so.

Another side of the template for analogy between mathematics and moral Intuitionism will be generally understood in the further argumentations as appeal to the concept of non-inferential propositions. There are two major approaches to expressing the understanding of what basic moral knowledge is: “self-evident” propositions (Audi, 2008) and intellectual “seemings” (Huemer, 2005). Both views have their differences and subtleties. However, the optic of thinking by analogy does not seem to be fine-tuned, at least for now. Further, we will put both approaches under one umbrella of basic moral non-inferential propositions understood as propositions, the truth of which can be known upon careful inspection of their content. ‘If mathematical truths are conceptual truths, and if we are to understand moral knowledge as being on a par with mathematical knowledge, then it must be the case that moral truths are also conceptual truths’ (Lutz, 2015, p. 45). Of course, one who rejects one or both premises, i.e. “mathematics is built on a set of conceptual truths” and “moral Intuitionism is worth taking seriously”, can find the further arguments begging the question.

Some additional information related to the structure of the dissertation is appropriate. The research question was posed in my early work (Korchevoi, 2023a). Thus, the above introduction is a reiteration of the previous discussion. Also, the 5-th chapter of the dissertation is dedicated to consideration of objections against ethical-mathematical analogy, in particular its practical implications considered previously in Korchevoi (2023b, 2024). Partially, those arguments are reiterated here. However, a detailed review of the philosophy of mathematics, methodology of the method of cases and analogical reasoning is presented here as new analysis. The review of the current situation with epistemological skepticism here is wider than previously. The main argument proceeds as a careful comparison of objects in the analogy; it's also new analysis. Finally, the consideration of metaphysical objections and premises bears novelty in this dissertation and provides prima facie plausibility of the conclusion.

2. Literature review

The following chapter analyzes an extensive number of sources in a particular order. First, we scrutinize the Method of cases in Analytic philosophy as an umbrella for the whole discourse of the dissertation. We aim to establish credibility of this approach and proper restrictions for its applications. Second, we analyze several different approaches to the philosophy of mathematics with the purpose of avoiding the influence of implicit assumptions in our main argument. Third, we provide an overview of current debate in epistemology related to Skeptical hypothesis and several implications for Moral epistemology.

2.1. Is Method of cases in Analytic philosophy in crisis?

This research is concerned with epistemology; in particular, with Moral epistemology. It uses the method of cases as a prime tool for carving out its conclusions. Thus, we find ourselves in the realm of Analytic philosophy. In the broad sense, philosophical inquiry is an attempt to understand whether an entity, phenomenon, event etc. has a particular property. In the realm of Analytic philosophy, an epistemologist asks: what does it mean if "a subject S knows that proposition p"? The epistemologist is concerned with propositional knowledge. The method of cases proposes a scrutiny of different situations where the subject S may find herself. Sometimes those situations come from the real world but more often they bear imaginary characteristics of a thought experiment, e.g. Trolley problem when one is deciding whether she can push a fat man from a bridge for the purpose of saving five innocent lives (Bedke, 2010). The corpus of literature using the method of cases is enormous (Zagzebski, 1994). Yet, there is a critique of the method from a metaphilosophical point of view (Baz, 2018).

A metaphilosopher can ascribe two assumptions to the method of cases (Baz, 2018). First is an existence, per se., of some answers when one extracts from the reality a particular situation and puts it in some theoretical context. Metaphilosophical attack on the method is that there are no answers to the important questions if we frame them into dry and artificial theoretical context. Second line of the attack is the objection that even if there are some answers, they are stripped of the meaning or usefulness for our real life. I can't help myself but note, following the irony of Cappelen and Deutsch (2018), that a metaphilosopher, while refusing to give credit to the method of cases, employs it by taking for granted the existence of a unified body of Analytic philosophy and ponders whether the Analytic philosophy has a property of "using a unified method of cases". Consequently, we can make two observations. First - it may be the case that metaphilosophy uses an optic which zooms out too far from the object of interest, i.e. the method of cases. A metaphilosopher does not find the method to be relevant because he doesn't find it on the philosophical "map". Second - in a broad sense - everyone, even metaphilosopher, is doing Analytic philosophy using the method of cases when trying to put her thoughts on paper. One may note that the second observation is an analogy of Kuhn-Popper tension in the philosophy of science. One may spend a good deal of time pondering whether a paradigm shift has happened in the Kuhn's sense in some particular area of science (Chen, Andersen, and Barker, 1998); yet when the person deploys her arguments in some coherent order related to the above shift, she does it bearing in mind her own fallibility and, more broadly, scientific fallibility in general, following the Popperian hypothetic-deductive thinking (Corcoran, 1989). Thus, for example, we have such peculiar problems as the Preface paradox (Jacquette, 2008).

Let us now turn to the substantial critique of the merits of Analytic Philosophy. It is important to note that the current analysis is concerned with epistemology. Therefore, it is wise to narrow down the debate about the criticism of Analytic Philosophy to the matter of epistemology. In doing so, we can see that the above first claim - when one extracts from reality a particular situation and puts it in some theoretical context, she is not in position to obtain valuable answers - is rather nonsense. Epistemologists felt pretty good for several millennia defining necessary and sufficient conditions for "S knows that p" following the definition of Plato's dialogue Theaetetus (Nawar, 2013). Since Plato, they have defined knowledge as justified true belief (JTB) until Edmund Gettier (1963) proposed a couple of cases which spawned a long and productive discussion among epistemologists. What epistemologists have learned from Gettier-like cases during the last six decades?

They came to understand that JTB is the only necessary condition for knowledge. Also, they have demonstrated that sufficient conditions for “S knows that p” are the matter of interplay between ability and anti-luck epistemological intuitions. The former intuition tells us that knowledge should be attributed to the cognitive activity of a subject in a substantial sense; the latter shows that knowledge cannot be a result of some coincidence of lucky circumstances. The analytical polishing of sufficient conditions is still in progress. It may be the case that epistemology will not find all the needed formulations in precise, short, and beautiful forms. However, to say that epistemologists do not have any answers would be nonsense. We have mentioned the term “intuition” here. It should be noted that by intuition we mean a special kind of epistemic intuition, i.e. ‘clear epistemic intuitions arise when the subject’s described condition plainly appears to fall on one side or the other of some significant divide in epistemology, such as the divide between knowledge and ignorance’ (Nagel, 2007: 793). We will develop the significance of the term “intuition” later in this chapter but for now it is enough to say that epistemology certainly does have, if not all, at least some answers due to the case method.

Now we shall consider the second line of criticism of Analytic Philosophy, i.e. that if there are some answers obtained due to the method of cases, those answers are fruitless for real life situations. This claim seems to be generated by one's feeling that sometimes an epistemologist's inventive creativity goes too far, deploying some bizarre cases. And it is certainly the case. When a reader encounters a question of how she can be sure that she is not a victim of “brain-in-a-vat” scenario or a “fake façade country” scenario, she is right to feel bewildered. Thus, it is the responsibility of the epistemologist to show that those cases aren't just sick mind games. Indeed, the Skeptical hypothesis does not try to convince us that we really can be brains located in some sort of mechanical boxes with electric connectors mimicking all of our everyday experiences. A skeptic asks us how we can be sure that we are not. And if we are not sure, then why we are not epistemically paralyzed by the above premise. One can claim that there should be precise restrictions for epistemological Skepticism to be viable, another may hold an opinion that mere supposition of subjective indistinguishability of our everyday experience from an altered one is enough to pose a threat (Beebe, 2010). In any case, the Skeptical hypothesis tells us that there are some situations when you cannot distinguish whether your perceptions and judgements follow from some sort of solid reality or form a mere illusion. For example, a wide realm of conspiracy theory is built on such thinking. Note, those who believe in conspiracy theories often take a refuge in asking: prove that I am wrong asserting that “aliens took over our government a long time ago”. As it happens, it is not an easy task for a skeptic’s interlocutor because the starting skeptic's premise is indistinguishability, by definition, of the normal government from a government “taken over by aliens”. Of course, one may simply ignore those silly conspiracy theorists. Yet, the skeptical threat does not disappear. It is silly to suspect that there maybe was a cosmic event yesterday that took away all railway stations from Earth but left all other features of our lives untouched while one is rushing to catch a train. It is not silly at all for a doctor to experience doubts while performing a differential diagnosis of a patient because, by definition, the differential diagnosis is an activity where it is difficult to distinguish the causes of a patient's symptoms. The former situation is a bad, bizarre example of the method of cases; the latter is legit and important. So, if questions posed by an epistemological skeptic have real life implications, does epistemology have any answers? Indeed, it may have some answers. For example, a doctor can be satisfied with epistemological Contextualism understanding that different contexts provide different degrees of epistemic responsibility for a subject. Thus, the doctor should be precise and cautious while making a diagnosis for a patient's life is dependent on whether the doctor knows that proposition p is true. Professional standards are different for different experts exactly because of different responsibilities including epistemic ones. Thus, we were able to refute both of the metaphilosophical attacks on the method of cases though not without paying a price. The price is an obligation for epistemologists to keep in touch with the ground of everyday life while theorizing about abstract issues.

Now we should consider why epistemologists claim that they are not losing an objective epistemological ground while applying the method of cases. It brings us to the discussion of epistemic intuitions. Epistemic intuitions in general are of particular importance for this study because they serve as an anchor not only for the method of cases but also for Moral Intuitionism per se. The expression “epistemic intuition” is used to label any immediate, not explicitly inferential ‘assessment of any claim of interest to epistemologists’ (Nagel, 2007: 793). We noticed above that epistemic intuition occurs when a case “appears” for an epistemologist to be an instance of, for example, S knows/not knows that p. In this way epistemic intuition bears resemblance with perception of visual appearance. It comes without such deliberate cognitive work as, for example, making a chain of proper deductions. Deductions may come later as means to check the intuition in relation to some epistemological theory. Consequently, epistemic intuition is afflicted by the same set of shortcomings as visual appearance, e.g. it can give us a misleading sense of being pro or being contra an existing epistemological theory. Thus, not every intuition is of use to us.

Historically, there have been two approaches to make use of epistemic intuitions in the method of cases. The first is called Reflective Equilibrium; it is associated with the name of Goodman (1965). ‘Applying this method in epistemology, we would start with clear examples of knowledge and produce a definition of knowledge that will help us to classify new or unclear cases; subsequent reflection on these new particular judgments is to help us refine our definition’ (Nagel, 2007: 794). The second approach was proposed by Carnap (1962); it is called Explication. ‘The process of explication starts from examples of current usage, or judgments of particular cases, but once a scientific definition is formed it is not subject to further constraint from reflection on the intuitiveness of its application to particular instances’ (Nagel, 2007: 795).

There are a lot of different avenues to critique the usefulness of epistemic intuitions. We are unable to track down all of them because the issue calls for a separate dissertation. Yet, even if the following paragraph may oversimplify the criticism, we address a conceptual umbrella for the criticism - a deep disagreement among all interested parties, i.e. philosophers and wider audience. One can argue that epistemic intuitions ‘lack the power to rationally persuade other professional analytic philosophers’ (Mizrahi, 2022: 969). Others may point out that among a wider audience there is also a disagreement. In the experiment conducted by Weinberg, Nichols, and Stich (2001), a sample of Rutgers undergraduate students was chosen; they were asked to identify whether a person named Bob possessed knowledge in a particular case. ‘Only 26% of those who had identified themselves as culturally Western saw Bob as having knowledge here, 57% of East Asian participants and 61% of Indian Subcontinental participants circled “Really Knows”’ (Nagel, 2007: 799). The roots of all the above disagreements seem to unfold as interplay of two crucial features of the method of cases: context-sensitivity and subject-sensitivity. We already briefly considered the issue of context-sensitivity which can be, at least partially, tackled, for example, by Contextualism. The issue of subject-sensitivity is a rather more complex matter. The subject-sensitivity unfolds alongside several disciplines in Humanity, e.g. linguistics, psychology, anthropology, sociology, etc. Given the above complexity, is our appeal to epistemic intuitions doomed? It is not, at least not completely. The solution is to look for a “universal core” of epistemic intuitions. Weinberg, Nichols, and Stich (2001) have ‘discovered that some questions produce the same reactions’ (Nagel, 2007: 801) across all the groups with different cultural and socio-educational backgrounds. As we will see shortly, the quest for the “universal core” epistemology is a rather ambitious project. What we will really need is to point out one epistemic intuition that is used by us irrespectively of our particular attitude towards the epistemological ground of our knowledge.

In order to clarify the usage of intuition in the method of cases, we consider a scheme proposed by Sękowski (2022). Using the method, we aim at clarification of a target concept X by asking whether a particular case is an instance of X. In different theories, different concepts are used as X, e.g. in epistemology X is “knowledge”. Sękowski (2022) identifies the above Goodman’s (1965) approach with the intuition of extension, and Carnap's (1962) approach - with the intuition of intention. What is intuition of extension? It is the same intuition as in the Theory of Reference. Sękowski (2022) notes the work of Kripke (1980) where the Causal Theory of Reference was formulated, i.e. a name refers to an entity by causality via its usage by a community of speakers. Kripke's proposal is a good tool for illustration of Sękowski’s (2022) approach though it may work for those who rely on other theories of reference. For the sake of simplicity, we can understand Kripke's proposal as such that we don’t talk in gibberish if we are engaged in a meaningful conversation. If by using a name we refer to an entity, the connection between the name and the entity is already established; thus, the issue of the scrutiny of the

Theory of Reference is to define the boundaries of the name's application, or its extension. As Sękowski (2022) puts it, while using the method of cases in the Theory of Reference, we ask whether the case in question is an instance of a name for an entity; we do not ask whether it is the reference per se. So, only the intuition of extension is used. The intuition of intention cannot be used in such a way that it bears our intentions to change the meaning of the verb "to refer". When we name, we refer; we do not talk in gibberish. Thus, the meaning of the operation "to refer" is set. Of course, it does not mean that a name's extensions are set too. Different communities of speakers can have different usages of words, or names. Thus, interactions of different localities of discourse can give birth to shifts in the meaning of particular concepts, e.g. race, gender etc. However, let us note that those shifts are likely to happen gradually, resembling a negotiation between communities of speakers at their own pace, if we intend to keep our speech as a coherent tool for communication. Yet, in epistemology, the situation is different. One may hold on to the conservative Goodman's approach supposing that the meaning of the verb "to know" is set and, consequently, she should be concerned with the definition of the necessary and sufficient conditions for the concept "knowledge", or an extension for the name "knowledge", by the means of inferential cognitive activity. This way, we discover what knowledge is. But it would be intellectually dishonest not to note that indeed, when the extension of the name "knowledge" shifts, so, in principle, may shift the meaning of the verb "to know " because of the causal connection between those two names. Thus, we arrive at the Carnap's approach where we use our intuition of intention. By intentionality here we mean our intention or willingness to change the meaning of the target concept "knowledge", or to change norms for it. Sękowski (2022) sees here a dichotomy Descriptivism vs. Normativity. In Carnap's approach, we indeed encounter the potential change of epistemic norms, therefore it seems reasonable to associate the approach with normative activity of a subject. Yet, we should not overlook the intentionality part. It is interesting how Sękowski (2022) uses the verb "to construct" repeatedly while analyzing the intuition of intention. I found it fascinating how persistently a particular question occurs in different areas of philosophy. This is the question related to the dichotomy of something being "discovered or constructed/invented". We will see later that this dichotomy gave birth to a whole new branch of constructive mathematics. Yet, now, luckily or not, we arrive at the central problem of this dissertation. Obviously, both Goodman and Carnap are pondering on norms for knowledge. While Goodman is sure that norms of epistemic enquiry should be discovered by the method of case, Carnap allows them to be constructed, at least partially. We want to tackle a similar question in Moral epistemology whether moral norms should be discovered or invented. However, we are not in the realm of Moral epistemology yet but in methodological analysis. Here the constructivists account is not radical. It would be unfair to say that Carnap excludes the intuition of extension from his scheme; his Explication begins from current usage of knowledge. This beginning is a matter of the intuition of extension. After that, the intuition of intention takes the turn in calibration of previously formed definitions of the term. Thus, both methodological approaches - Goodman’s and Carnap's - aim at the core epistemology from the beginning. The former hopes to polish it enough, the latter attempts to modify it for good.

As a concluding remark for this analysis, we can reiterate several points that will be used in further discussions. We have found that despite some current philosophical concerns, the method of cases can still provide informative answers for some epistemological questions, but not for all. The epistemological ground for the method is secured by at least one of the epistemic intuitions, namely the intuition of extension. If one would like to argue in favor of abandoning this intuition, she is likely to lose the ground for making meaningful references in her speech. Consequently, such a person would be reliving Ferdinand de Saussure’s bewilderment with the problem of psychological contingency in associating of the signified and its signifier in our language (Stoltz, 2021). Last note concerning the above analysis is that the Theory of Reference has its own complexities and disagreements despite that its target concept ‘is much more stable than most other concepts, examined in philosophical considerations’ (Sękowski, 2022: 22). In particular, analysis of fictional names and names for abstract objects is important for us. We will consider the latter in detail through the lens of the philosophy of mathematics in other chapters.

2.2. Review of philosophy of mathematics. Is mathematics discovered or invented?

We should state clearly what sort of mathematical epistemological attitude is involved in our arguments. Consequently, we should provide a more or less clear picture of what is meant by the utterance “by analogy with mathematical knowledge”. A reader from a wide audience is likely to ponder: why should one bother with the above topic at all? From the outside, mathematics seems to be a body of robust knowledge which is obtained by a “guild” of mathematicians in a somewhat mysterious way. Indeed, dare to name even one person outside the “guild” who can even very tentatively grasp the essence of Perelman’s proof of Poincare hypothesis (Hosch, 2024). Yet, such a position will give us nothing but shallow speculations in ethics.

If we ponder how mathematical knowledge can contribute to the speculations about Moral epistemology, we should not make two mistakes. First, there is no totally unified body of mathematical knowledge and, consequently, no unified body of mathematical epistemology underpinning it. The major divide goes along the line between pure theoretical and applied mathematics. As we will see, this divide entails quite different epistemological attitudes. But even in the realm of pure mathematics, there is a disagreement. Thus, one should not expect that we can take some mathematical knowledge as premises and deduce from them moral truths, building sound and valid arguments. Our best hope is the construction of a valid argument of the kind that if one agrees with such and such epistemology of some part of mathematical knowledge, then one should admit such and such consequences in Moral epistemology. Therefore, there is little hope to convert a moral skeptic who does not believe in reality of any mathematical abstraction. Likewise, our arguments will be powerless for a person who strictly believes that all math is a product of the human mind. If one is ready to discuss Kantian moral law in us, she at least must have no doubts about the existence of the stars above us. Further, we will discard several ramifications of mathematical nominalism, logicism, and constructivism on this basis.

Second, we shall not try to construct a sort of homomorphism, or structure preserving mapping, between mathematical and moral knowledge. In order to defeat the position that there is no moral knowledge at all, it would be enough to show that there is some moral knowledge. Therefore, we are not obliged to expose all moral knowledge; even less we are obliged to tame the projects of mathematical and/or moral epistemology as a whole. In such a way, we will be skeptical of defeaters of Moral Intuitionism that jump from one mathematical proposition to another without giving support to why those propositions belong to a proper epistemological domain. The speculation that both of them may have something to do with mathematics is not good enough. Arguments involving Russell-Zermelo paradox and "set of all sets that are not members of themselves" can be fascinating but unfair in our case. We are likely to use very basic blocks of mathematical knowledge, e.g. some understanding of Peano arithmetic and Euclid geometry. So, no special knowledge of mathematics is required. In this, I disagree with Dummett (1994) who asserts that while basic physics can be explained to anyone without turning to a domain of objects special for physics, math cannot. I suppose that any understanding of basic knowledge of some specified intellectual realm and/or activity involves basic understanding/grasp of the meaning of its objects. In order to get some understanding of Newtonian mechanics, one must grasp the meaning of the terms "a rigid body", "center of mass", etc. Likewise, one should grasp the meaning of a natural number, its successor, etc. to be in a position to understand arithmetic. The essence of such intellectual grasp is of concern for further analysis.

There is a variety of views among mathematicians and philosophers of mathematics about the origin of mathematical objects. No clear agreement exists among mathematicians about the debate of "discovery" vs. "invention" of mathematics (Ernest, 1999). Ernest (1999) describes two possible approaches: absolutist vs. fallibilist. 'The absolutist view of mathematics sees it as universal, objective and certain, with mathematical truths being discovered through the intuition of the mathematician and then being established by proof' (Ernest, 1999: 1). On the other hand, fallibilists assert that mathematics is a matter of evolution, it is invented and affected by changes; therefore, mathematics is fallible. A similar dilemma arises between logicians in the conversation about the existence of "first principles” in the Theory of proof (Corcoran, 1989). Corcoran argues that foundationalism in Logic is attributed to Aristotle, Pascal, and Frege ‘which presupposes universally knowable “first principles” as the ultimate premises for all propositions proved to be true’ (Corcoran, 1989: 23). He lists Russell, Tarski, and Popper as proponents of the opposite view called “probabilism”. Such naming can be a bit misleading. Well, who wants to be an absolutist who ignores the Popperian humility (Thornton, 2023)? Math is a science, not a dogma. Isn't it? However, the real and old divide in the philosophy of mathematics which reflects the debate of “discovery” vs. “invention” of mathematics is the debate between mathematical Platonism and mathematical Constructivism.

Now we shall carefully describe those two opposing approaches toward the substance of mathematical objects and the value of truth and proof in mathematics. In a nutshell, mathematical Platonism is based on the following three assumptions: mathematical objects exist; they're abstract; they are independent of any activity of an intelligent agent. Thus, existence, abstractness, and independence are the vectors to attack mathematical Platonism. It seems that defense of the abstractness is the least problematic. Dummett (1994) notes Frege’s example of the term “equator”. It is abstract; no one will feel anything crossing the equator, yet it is objective. Same goes to many conventional concepts: law, corporation, humanness, etc. The way of cognizing of such concepts is noteworthy: ‘reference to an abstract object is to be understood only by grasping the content of sentences involving such reference’ (Dummett, 1994: 22). We may discard from the consideration the rivals of Platonism who fear the abstractness per se., e.g. such nominalist ramifications as fictionalism (Field, 1980). So, the real problems begin with the need to establish the existence of mathematical objects. Dummett (1994) rightly points out that many abstract objects exist but their existence is somewhat contingent. The existence of the equator is contingent upon the existence of physical objects and relations among them. If it occurred that we lived on the “flat” earth, the existence and meaning of the term “equator” would be doubtful. Yet, we expect that mathematics explicate necessary truths. Thus, ‘the significant distinction is not between abstract objects and concrete objects, but between mathematical objects and all others, concrete or abstract’ (Dummett, 1994: 24). However, if we assert the existence of mathematical objects and a set of non-contingent necessary truths about them, independence falls under a threat immediately. As such truths should be in no causal connection with physical reality, how can we have epistemological access to them? If there are enough mathematical objects and mathematical truths to fit the scientific description of physical reality, then is this phenomenon contingent per se.? So, should Platonism be refuted? There are two ramifications of Platonism: one containing robust independence, the other - weak, or counterfactual, independence. Robust independence claims that mathematical objects are on par with physical objects. Counterfactual independence is a weak condition demanding that mathematical objects would exist even if there had not been intelligent agents. It is very likely that Dummett (1994) refutes the robust independence in Platonism by saying that we cannot be sure in coherence, and even consistency, of the corpus of mathematical knowledge, 'if mathematical structures are merely the inhabitants of another realm of reality, apprehended by us in a manner analogous to our perception of physical objects' (Dummett, 1994: 20). Thus, robust Platonism seems to employ too crude an analogy between our world and the mathematical one to catch the subtitles of both. Yet, there is hope.

The weak counterfactual independence with the abstractness and the existence can be redeemed through Fregean logicism (Frege, 1953). Logicism in the broad sense is a doctrine of logical foundationalism extended to mathematics; considered together with the assertion of existence of natural numbers, it is not in contradiction, if not coincides, with weak independence Platonism. Fregean logicism 'explains various puzzling features of mathematics. It explains its methodology, which involves no observation, but relies on deductive proof' (Dummett, 1994: 21). It explains our intuition about generality and the necessity of mathematical truths. Yet, 'the aim of representing a mathematical theory as a branch of logic is in tension with recognizing it as a theory concerning objects of any kind' (Dummett, 1994: 21). Generally, logic is concerned with truth-values of propositions consisting of terms and connectives. Thus, while applying logic to mathematical objects, we must be able to specify to which domain of objects a particular term is applicable. When a term is applied to a concrete object, it is enough to specify a criterion of application and a criterion of identity. As humans already have some “grasp” of what concrete objects of physical reality are, the term's applicability is a sort of refinement of this grasp. A different story unfolds in the case of mathematical objects. 'For a term of this sort, we make a further demand: namely, that we should "grasp" the domain, that is, the totality of objects to which the term applies' (Dummett, 1994: 25). Russell and Whitehead's project of logicism attempted to turn around this difficulty by preserving independence of logic from the existence of any particular objects. Russell and Whitehead counted mathematical objects as surrogates, or even place-holders, inserted in propositions. Unfortunately, in order to build a consistent mathematical theory, they were forced to introduce assumptions, e.g. Axiom of Infinity, which were neither undoubtedly true nor belonging to the realm of logic. Frege believed in the existence of natural and real numbers; thus, consequently, he believed in the logical truth of number theory. And for a while, he seemed to be secure in his convictions until Cantor's diagonal argument came about. Cantor showed that 'the totality of real numbers is not denumerable' (Dummett, 1994: 27). So, since Cantor, we know that there are different cardinalities of different infinite totalities. If one is honest enough, she/he must admit that this somewhat ruins the robustness of one's utterance: "I have grasped the totality of real numbers". For the sake of simplicity, we omit another powerful strain of arguments related to the paradox of "a set of all sets that are not members of themselves". At this stage, we find ourselves in somewhat anecdotal position of an Ancient Greek philosopher, e.g. Pythagorean, thinking that infinity, as a cosmic villain, got us finally. As a way out of this bypass, Dummett (1994) proposes a concept of an indefinitely extensible domain. Real numbers is an example of a such concept. Dummett also counts natural numbers as such a concept because in order to grasp the totality of natural numbers, one needs to go out of the domain of the totality: every natural number is finite but the cardinality of natural numbers is not. I would disagree with Dummett here because our problem was with an ability to "grasp" the totality. Natural numbers demand of us to be comfortable with the calculus of limits, real numbers - to be comfortable with the calculus of infinitesimals. The former may come naturally and intuitively to many, the latter is a much more complex matter. Can one honestly say that she/he has never struggled with the task of "taking the integral"?

Thus, if we take seriously Dummett's proposal to comprehend numbers as a concept of indefinitely extensible domain, then ‘this fact compels us to adopt a thoroughly constructive version of analysis’ (Dummett, 1994: 28). We are unable to comprehend a real number but only its approximation, therefore, a constructive theory of real numbers fits better ‘to their applications than its classical counterpart’ (Dummett, 1994 :28). Now we shall turn to the consideration of mathematical Constructivism. However, let us not forget in the further argumentation the critical characteristic given by Dummett’s word “application”, likely entailing the degradation of the boundaries between pure and applied mathematics.

We are going to focus on mathematical Intuitionism (Brouwer, 1975) as a keen example of the application of constructive apparatus to mathematics. Mathematical Intuitionism asserts that mathematics is a product of a mind and mathematical statements are proven via mental constructions of the mind. We suppose that crucial distinction between different veins of Constructivism rests in the realm of metaphysical issues, i.e. what sorts of minds are involved in the construction of mathematics. Though one may find that this is an oversimplification from a point of view of a mathematician, moral epistemologists should keep a balance: it is crucial to explicate an epistemological modus operandi of a mathematician without claiming to be one. Brouwer starts the building of mathematical Intuitionism with admitting the reality of one Kantian-like a priori contemplation: time. Thus, a mind is able to distinguish two moments of time falling apart. In such a way and in this moment, we distinguish one of something -ish from two of something -ish, or “twoities” in Brouwerian words. It is the first act of mathematical intuition. The second act is the ability of a mind's intuition to create infinite sequences of mathematical entities and/or their properties using entities which were constructed previously. So, from “one” and “two” one constructs “three”, etc. Thus, in mathematical Intuitionism, truth and falsity of a mathematical proposition, formula, theorem are temporarily dependent: if a mathematical statement is proven at the moment t, it will remain so. But before this moment, it lacks truth-value. As we noted above, the important issue is whose mind is the creator. Brouwer introduced a concept of Creating Subject as an ideal mind. Other veins of Constructivism are satisfied with existence of a mind of a mathematician whose work goes from production of mathematical proofs in ideal circumstances to cognitive activity of an average fallible human being (Ernest, 1999).

Now we shall put forward two important arguments of why we are going to focus further on Brouwerian mathematical Intuitionism and discard other ramifications of Constructivism. I will call the first argument Bizarre Anthropocentrism. Though we have moved far from Platonism, the intuition about the weak independence of mathematics from the existence of species living on a tiny planet located in a far corner of not the biggest galaxy is hard to refute completely. If there had not been humans, would there be mathematics? If there were non-human intelligent agents, would they build/construct different mathematics? If their mathematics were different, i.e. no “translation” of mathematical sentences from our math to an alien one is possible, would this different mathematics fit the reality, given that they live in the same Universe? If it would, then why, for God’s sake? Note that the burden of proving the soundness of the view that math is a product of the human mind lies on a person who asserts it. So, my reasonable move would be to consider an ideal intelligent agent. An instantiation of such an agent can be made by an approximation of a human mind work but the instantiation isn't equivalent to the idealization. It seems that Brouwer did exactly this with the concept of Creating Subject. Another way out for a proponent of the above anthropocentric position would be to say that she doesn't care about metaphysics at all. At the end of the day, the application of Constructivism will hold irrespectively of the answers to my weird “alien -ish” questions. Here, we arrive at the second argument contra the need to debate with a wide range of Constructivism which I will call No Proselytism. As our purpose was to refute a malignant skeptical moral argument that there is no moral knowledge in the sense that this knowledge can be more than human mere rationalizations of survival of the fittest, we are unable to provide a convincing argument for a person who is convinced that there are no truths that go beyond the capacity of human mind in the first place. If one is able to suppose that there are universal truths, at least in some intersubjective sense, then we can try to borrow some epistemological solid ground from mathematics and, thinking by analogy, give this power to Moral epistemology.

Nothing comes without a price. So, what Brouwer had sacrificed in the process of developing the project of mathematical Intuitionism? First, the project demands to abandon classical logic. In particular, the Law of Excluded Middle (LEM) is not valid any more. In order to assert that (not-A V A) is true, one should construct a proof that A is true or provide a construction that shows falsity from any possible proof of A, i.e. prove that not-A. Before the moment of exposure of such proofs, the truth-value of the above disjunction is unknown. Thus, mathematical Intuitionism gives birth to modern Intuitionistic logic, or Brouwer-Heyting-Kolmogorov interpretation of logic (Plisko, 1988). Though one may feel angst leaving the classical logic behind, we find some solace in knowing that effective mathematical apparatus can be built using non-classical logics. The same applies to Moral epistemology. For example, the behavior of the negation operator in Moral epistemology is not so simple. Consider the proposition that “killing is moral”. One may suppose that the proposition is false. Then its negation “it is not the case that killing is moral” should be true. Is it though? Now consider the proposition that “killing is immoral”. One still may be inclined to count the proposition as true. Yet, “it is not the case that killing is immoral” can bear some sense of truthfulness for some people. Would you blame a victim who, by accident, has killed a psychopath and serial murderer? Further, while one may agree that killing is immoral, “not-killing” probably is neither moral nor immoral in most situations, but not in all. It can be just normal. So, while “not-killing” can be neutral, “killing” always seems to be morally laden. Of course, here we meet such issues as contradiction vs. contrariety, usage of universal quantifiers and negation, modal analysis of negation, and even morally vague predicates. We will consider those questions in more detail in the further chapters. For now, let's note that LEM application is not always straightforward in Moral epistemology and there are approaches to mathematics that also consider LEM as a matter for intellectual inquiry.

Some other features of Brouwer’s approach are quite interesting for our consideration. His second act of intuition allows the Creating Subject to construct sequences of mathematical entities, in particular “law-like” and “law-less” sequences. The former are sequences obeying any kind of law, or algorithm of construction. The latter are sequences that are constructed using the Creating

Subject's free will; thus, they may bear a pure contingency, randomness. It is quite a fascinating consequence given the metaphysical issue of deterministic reductionism. Could a human produce an algorithm such as a generator of random numbers without the involvement of any physical phenomena, e.g. rolling a dice? It is not a trivial problem of our ability to be sure that something is a product of our mind per se., and, simultaneously, of our free will. It also raises a question about the close connection between the concepts of randomness and free will. However, the further discussion about the essence of the concept of Creating Subject among mathematicians is of particular importance from an epistemological perspective. 'Kripke and Kreisel in the mid-1960s encouraged further discussions on Brouwer's arguments by presenting explications in terms of what have become known as Kripke's Schema and the Theory of Creating Subject' (van Atten, 2018: 1569). Below, we provide the Kreisel's axiomatization of Creating Subject. Let's denote operator □ n A as "Creating Subject has a proof of formula A at the moment n". Thus, (1) □ n A V not-□ n A, (2) if □ n A then □ n+m A, and (3) A iff exists n such that □ n A. The axiom (1) means that the Creating Subject can decide whether he has proof of formula A at the moment n, axiom (2) means that he has a memory of the proven formula, and axiom (3) means that the Creating Subject proves only true formula and if formula is true, then it will be proven. The last axiom is the most controversial; its formulation considered here is due to Troelstra (1973). For now, we note that a serious discussion took place among mathematicians. Interestingly, some of them designated themselves as constructivists, simultaneously allowing the possibility to formalize an ideal mind via axiomatization.

At this point, one can be satisfied with the results of our investigation of the epistemological ground of mathematical knowledge. We have found that many, if not all, mathematicians probably hold some beliefs prior to their mathematical inquiries. Platonists believe in the existence of abstract mathematical objects and mathematical truths. Logicists believe in the existence of some abstract mathematical objects and logical truths about them. Mathematical Intuitionists believe in the existence of some ideal mind and our ability to grasp this idealization. Others, e.g. Nominalists, social Constructivists, etc., are holding the priority of human mind activity in the mathematical inquiry. Yet, if we are unable to tilt their priorities by presenting the Bizarre Anthropocentrism argument, then it is less likely that we can save them from moral relativism, or even nihilism, through unfolding epistemological ground for moral knowledge by analogy with mathematical knowledge. However, one more step is needed because one may wonder whether an ideal mind dwells in the realm of ideas and, if so, what those ideas are. In other words, we need to understand what epistemological price would be paid when one leaves behind the realm of abstract mathematical objects and embraces the epistemology based on an ideal mind.

Our prime objective is to deal with the first crucial challenge for epistemology: the Skeptical argument. We will consider Skepticism in detail later. However, there is also the second challenge which is the Gettier cases. Since Gettier (1963) wrote his paper, we came to understand that JTB template is the only necessary but not sufficient condition for knowledge. The problem arises because of a reasonable presupposition that knowledge cannot be a matter of luck. Some examples of well-known Gettier cases are provided below.

For the sake of a thought experiment, let us suppose the following. One came to believe what time it is by looking at the clock in one’s kitchen. Usually, this is a very reliable clock. Yet, the clock stopped and one came to the kitchen one morning at exactly 9 o'clock. Suppose the clock stopped exactly twenty-four hours earlier and the person did not know about it. Therefore, one can have a justified belief that it is 9 o'clock by looking at the clock. Does the person know what time it is? One cannot know the time by looking at a stopped clock. Thus, it is just a matter of luck that one’s belief is true (Bogardus, 2014). Here is one more example of a Gettier's case. A farmer looks onto a field through the window. He sees what looks very much like a sheep and he forms a proposition: “there is a sheep in the field”. Nevertheless, it is not a sheep. He is looking at a big hairy dog. By luck, it happens that at that moment, there is a sheep hidden from a farmer's view behind the big hairy dog. Does he know that there is a sheep in the field? The corpus of cases produced by epistemologists since Gettier's article was published is enormous (Sosa, 1999; Pritchard, 2017). It contains some bizarre cases, e.g. catching an image of a real barn by a traveler in a fake barn façade country.

There is a general scheme for constructing a Gettier case. One takes a belief which is justified but the current situation makes the belief false and adds to the case a matter of luck, which makes the belief true (Zagzebski, 1994).

The threat the Skeptical argument poses seems to be inescapable in general. Mathematics can be afflicted by Skepticism in principle; yet, in the later chapters we will show that the degree of the threat depends on the particular attitude toward realism of the abstract mathematical realm. Now, the important question is the following: can a mathematician also be a victim of a Gettier's case?

Pritchard (2017) sees knowledge as an interrelation between two intuitive conditions. The first is the anti-luck intuition: one's true belief is not a matter of luck. The second is the ability intuition that is 'one's cognitive success is attributable in some significant way to one's cognitive agency' (Pritchard, 2017: 59). The precise, subtle game of the above conditions for knowledge is beyond the scope of the text. It would be sufficient to note that epistemologists systematically try to spouse the ability condition with the "safety condition". The safety condition is a family of theories that count that one's true belief has to be, 'in some important sense, safe' (Bogardus, 2014: 289).

Let us return to the term "mathematical knowledge". Some mathematicians unfold the term as containing an expert's knowledge that 'is the knowledge possessed by an active qualified practitioner' (Corcoran and Hamid, 2014: 5). The expert's cognitive processes, or "know-how", seem to be crucial for obtaining knowledge, including mathematical knowledge. Indeed, it seems to serve as a proper example of the above ability condition for mathematical knowledge. 'In fact, the hallmark of the expert is the ability to call to mind open questions, or hypotheses, propositions not known to be true and not known to be false' (Corcoran and Hamid, 2014: 5). Unfortunately, it does not answer the question whether mathematical knowledge is "safe" in the above epistemological sense.

If we consider the general scheme for the Gettier case (Zagzebski, 1994), then we can note that the matter of luck that inflates Gettier cases is unknown and, therefore, at least partially, independent of the agent. Moreover, there is almost no blame on the agent because her/his justification has to be reliable, tested many times already. It is a particular situation that exhibits a compensating effect of good and bad epistemic luck. Also, despite some oddity of Gettier's examples, epistemologists are concerned with the problem because they see that it can penetrate into our everyday life easily. The case of catching a glimpse of an image of a real barn by a traveler in a fake barn façade country can be reiterated. Let’s say a person goes to a car dealership and sees ten used Hondas. She did her research. She is convinced that Honda is good. So, she picks up a car and it really is good. However, unbeknown to the person, the rest of the used cars in this dealership is complete trash. Such everyday situations raise epistemological concerns. Gettier's situation is all around us and yet we know a lot. We navigate through our lives without stopping every minute to experience epistemological bewilderment. Thus, if we want to assert that a mathematician can be “gettiered” (such term occurred in epistemology after a couple of decades of scrutiny of the issue), then we should provide some simple and elegant examples of a mathematician having a good mathematical justification of a true belief, yet, failing to obtain knowledge.

In the case of applied mathematics, we have to admit that applied mathematical knowledge is likely to be afflicted by epistemic luck. If one comes out with a true theory but a practical application of the theory demands a use of some tools, then there can be situations analogous to the above case of the broken clock. One obvious example is a calculation made with a weird broken calculator that makes one mistake but compensates for it with another. The idea is obvious and it can be garnished by substituting the calculator with a complex program and a computer (Barton, 2023). Yet, there may be an objection that the broken clock case tells us about the engineer that is “gettiered”; the applied mathematician is safe. Thus, we should focus on the contingent features of the real world to which a justified mathematical theory is applied. Let us suppose that a cyclist plans a trip. He applies a good mathematical theory to assess the length of his path. However, unbeknown to him, a renovation of a bridge is on his way. At the moment of planning the trip, a part of the bridge for cars is completed and it has a serious elevation and curvature unknown to the cyclist. So, his estimation of the path is false. However, on the day of his start, the route for bicycles was also completed and it had no elevation exceeding the characteristics of the bridge before renovation. Thus, his estimation of the length of his path turned out to be true. This case seems to lean less on the mechanical errors of the mathematical theory application than the broken clock case. Still, one may object that the problem is not with the math but with the map. Well, we can consider a set of so-called probabilistic paradoxes. They aren't paradoxes in the strict sense but rather illustrations of difficulty for our common sense to grasp probabilistic analysis. Let us give an example called St. Petersburg Paradox. There is a game for which, if you want to play, you should buy a ticket. The game goes like this: a fair coin is repeatedly flipped. If tails is obtained, the game is terminated. If heads is obtained, the sum of money in the basket which you can take is doubled. The game starts with one dollar in the basket. How much are you willing to pay for the ticket? One mathematical strategy is to assess an expected value which is a sum of multiplications: probability of an outcome multiplied by the value of the outcome. The expected value is infinite. Does it mean that it is rational for a person to pay any amount of money? The analysis can be made from another angle. Your win is a consequence of several consecutive heads. For example, the probability to get five consecutive heads is 1/32, which is about 3%. It is not too high a probability. Even if such an event happens, the winner gets 16 dollars. So, despite the infinite expected value of the game, it is not worth playing. As Peters and Gell-Mann (2016) note, 'expectation values are only meaningful in the presence of ensembles or in systems with ergodic properties' (1). However, the question of detection of ergodic vs. non-ergodic stochastic processes is hard to implement into mundane decision-making strategy as it entails an assessment of non-trivial behavior of different runs, or ensembles, of the stochastic process. Yet, one could describe a Gettier-like case by supposing that a person made her decision based on the expected value, bought a ticket for a thousand dollars, and by sheer luck got 100 consecutive heads thus winning millions. In this situation, we did not imagine odd events in the physical world but supposed an occurrence of an improbable but not impossible event. Consequently, it is reasonable to assume that in the situation of applying a mathematical theory as an approximate model for a part of the real physical world, some epistemic luck may afflict one's epistemic ability.

The case of pure theoretical mathematics seems to be different. One of the first proposed approaches to deal with the Gettier problem, in general, was "No

False Lemmas'' that is a hypothesis that the following 'condition would do the trick: S's belief that p is not inferred from any falsehood' (Ichikawa and Steup, 2014: 7). This approach does not fulfill the epistemological expectations in general because it excludes a wide range of every-day knowledge. Usually, we do not try to catch smuggled false premises in the process of navigating through our daily routine. We already noted the phenomenon of having a lot of everyday knowledge despite the closeness of Gettier's threat. Wouldn't it be odd if every time one is hoping to catch a train, she would check if there was an accident, strike, earthquake etc. preventing proper work of transport? Yet, in the case of weak mathematical Platonism or Fregean Logicism "No False Lemmas" condition, will do the trick: a proper application of expert's skills allows the transition of the truth from true premises via valid deductions to a true conclusion. But what about Constructivism? Is it immune from Gettier's threat?

First of all, if we treat some extreme forms of mathematical Constructivism, e.g. social Constructivists account, as a blueprint for mathematical knowledge, then immediately we realize that often 'a fallacious chain of reasoning is found in an argumentation whose conclusion actually follows from its premise set' (Corcoran, 1989: 26). We are trying to take a look at mathematics from an epistemological point of view. Let us consider a recent article by Barton (2023) who looks at epistemological problems through a mathematical perspective. As we noticed above, Barton (2023) provides some Gettier-like arguments for applied mathematics though not without shortcomings. He claims to do the same for mathematics per se. Yet, he doesn't. He starts from an assumption that mathematicians are fallible and, therefore, often they provide proofs which can be a matter of further refinement. So far it is reasonable but only if he is employing social Constructivism. Indeed, if one uses as a criterion for good mathematical justification a process by which one's mathematical arguments are 'shared and checked by the rest of the community' (Barton, 2023: 3), then it is possible for one to get confused if she mixes epistemic luck with fallible human cognitions. However, this luck should be attributed to a fallible mathematician but not to an epistemically hostile mathematical realm. As a result, all Barton's (2023) examples are clumsy excuses for mathematicians to make mistakes. So, the epistemological response is not to make them or to amend them if they are made. At the end of the day, mathematics is the most charged human activity with the intuition of the necessity of its results; therefore, it puts some professional demands on its practitioners. Note, that our further critique of Barton's claims witnesses rather the non-obviousness of the question of a "gettierd" mathematician than our desire to overthrow his epistemological attitude. Barton (2023) defines the source of "gettierd" mathematical theories as 'non-trivial gaps and hard-to-detect errors' (13). This definition seems to be exemplified by two phenomena: examples from the axiom selection and the black box lemmas. Let us not forget that here we are interested in the formulations of clear examples from mathematics. So, Barton (2023: 16) demonstrates an example from the Set Theory noticing that from one set of axioms follows the Projective Determinacy statement but from another, a modified set of axioms, follows the truth of Continuum Hypothesis. Immediately, he says that his claim that one set of axioms is better than another entails strong set-theoretic realism. So, how can a reader reconcile Barton's social constructivism with the set-theoretic realism? Is this a sort of partial mathematical empiricism that holds that the Set Theory is an essence of all mathematics and there is only one true Set Theory and its axioms flow from reality of the physical world? If so, do they bear necessary truths? If the axioms are real and abstract, then what are the right criteria for the axiom's selection? Also, Barton seems to be uncertain about mathematical pluralism. Does mathematical pluralism have no rights at all or only in some fields of mathematics? Barton's position is not clear. Yet, until one provides a solid ground for why we should abandon the usage of Euclid geometry and Lobachevsky geometry, given that both theories are consistent in themselves and applicable to different geometric shapes and spaces, mathematical pluralism has its merits. The next source of prospective "gettiered" mathematics is black box lemmas. Those lemmas are somewhat hard to prove; yet, they are used by the community of mathematicians. They may even be false but some other lemmas, often their weak variations, can be proved instead of them. Thus, one may come to believe that theorem A is proven though it is not so. The agent himself or his colleague may correctly prove that theorem B follows from A and indeed B is true. Do they know the theorem B? Does one of them know B while the other doesn't? The case seems to strictly violate the "No False Lemmas" condition. But a proponent of some philosophy of mathematics has the right to ask:

why should we be allowed to apply the condition in the first place? Constructivism, at least in its robust forms, cares about constructive proofs.

Let us imagine a spectrum of proponents of mathematical Constructivism where on one side social constructivists are located and on the opposite side - mathematical Intuitionists. Social Constructivism is no better than any other human cognitive activity, therefore this approach allows unproven lemmas to influence our judgment. Though it is still unclear how an epistemic luck, independent from an agent, can penetrate the case other than through the testimonial knowledge. Now we will allow ourselves to move along the above spectrum toward mathematical Intuitionism. Somewhere we meet a mathematician who says that if a lemma is proved, we have a solid constructive proof and there is no space for it to be semi proved. At this moment, “No False Lemmas” condition is applicable. Now it becomes clear why Brouwer, Kripke, Kreisel and some others were in the position of need to give a formal axiomatization to the concept of Creating Subject. If we remember that the formally described Creating Subject knows what is proven, remembers what is proven, and proves only true formula, then we see that the condition “No False Lemmas” is trivially applicable to the results of Creating Subject’s cognitions. Thus, while it is not obvious that all of Constructivism can be saved from Gettier's threat, mathematical Intuitionism is defended well by the Creating Subject.

We arrived at a somewhat worrying point. Still, I suppose that the majority of mathematicians would say that they do better than a traveler through “a fake barn façade country”. Mathematics has, or should have, some solid epistemological ground. Yet, if one starts from idealistic underpinning of mathematics, she runs into the need to validate her ability to deal with totalities. If one begins to construct the math step by step, she quickly loses the solid epistemological ground and is forced to call for help of Creating Subject, the concept that is not easy to admit and digest.

Yet, as a preliminary conclusion of our journey into the realm of mathematical philosophy, we could say that the situation is not hopeless but it rather encourages us to dwell in a real epistemological realm. If one admits that in order to defeat radical Skepticism our best, so far, epistemological attitude is to commit ourselves to a belief in epistemological value of a small set of propositions, then so be it. Should the fate of a mathematician differ significantly from the fate of an epistemologist who tries to convince a stubborn skeptic to abandon the “brain-in-a-vat” trap? Remember Pritchard's (2016) idea that in order to make any rational inquiry possible we are likely to be bound to several fundamental epistemological commitments, or “hinge commitments”, that ‘are neither acquired via rational processes nor directly responsive to rational considerations in the way that normal beliefs are’ (10). For mathematics, such commitments can resemble a Fregean belief in the existence of natural numbers or the Brouwerian dependence on two acts of intuition. By the way, Pritchard supposes that to deal with Skeptical hypothesis, we will need only one commitment, i.e. that one is not a victim of radical Skeptical scenario. Thus, our doxastic realm most likely includes some propositions which we are asked to merely believe; this propositional corpus is pretty small. One can feel self-pity based on human epistemological restrictions or arrogance based on the position that humans are Alpha and Omega of knowledge acquiring process. Both attitudes are fruitless. Instead, it would be better to dig deeper into the essence and the epistemological ground of the fundamental commitments. Though Pritchard’s proposal was not too hopeful of exploration of epistemological ground of the fundamental commitments in general, the mathematical starting points can be highlighted in a clear manner with the concept of “self-evident” propositions. This concept will be a matter of one of the following chapters.

Despite having tried to shed light on the issue of the foundation of mathematical knowledge and reaching the above conclusions, I cannot help but to propose further one thread of arguments that pop up in the matter persistently. It is too metaphysical, almost religious, to be voiced. So, consider the following as only a poetic illustration. Paul Gordan, while having difficulties trying to construct the invariant theory, exclaimed that ‘it is not mathematics; it is theology’ (cited in Mandelkern, 2024: 2). Some theologians who are in love with mathematics can't help themselves but to notice that several universal concepts, for example “beauty”, essentially belong to both realms: theology and mathematics (Kessler, 2018). Let us do some theology then. The term “god” pops up here and there in the discussion of pro and contra constructive vs. non-constructive mathematics. The term can occur as a conviction of a mathematician, e.g. the well-known phrase of Leopold Kronecker: ‘God made the integers, all else is the work of man’ (cited in Mandelkern, 2024: 2). In addition, one can encounter the term in the consideration of metaphysical assumptions underlying physics, e.g. the assumption that magnitude of any quantity can be equivalent to a particular real number ‘known to God, if not to us’ (Dummett, 1994: 28). Let us consider the axiomatization of the Creating Subject and allow the time variable n to go to infinity. On one hand, it is clear that as a limit of the axiomatization, we obtain classic logic with LEM and the statement that mathematical truths are necessary truths. On the other hand, the concept of the Creating Subject amended by the ability of the Creating Subject to move at will along the time scale resembles a description of an omniscient creature. Thus, finally a metaphysical poetic contemplation can answer the question: is mathematics discovered or invented? It was invented by God. It shall be discovered by Platonists or re-invented by mathematical Intuitionists.

2.3. Current debates related to Skeptical hypothesis in epistemology

There are two problems that have been bothering epistemologists. We noticed the problem of Gettier cases already. Yet, our major concern is with the problem of Skeptical hypothesis. To draw a sketch of the problem, imagine that a powerful epistemological demon took out your brain and put it into a vat, simultaneously connecting the brain to wires in such a way that all the experiences and perceptions of your brain are indistinguishable from the actual world. Thus, though you seem to live in the actual world, you cannot know that you are not a “brain-in-a-vat”. Therefore, you cannot know that you have hands. This case is bizarre, yet, it does not violate the above restrictions of the method of cases. We can think about dreams or hallucinations as possible scenarios to link the Skeptical hypothesis to real life. The roots of this problem go deep into the past of philosophy, at least to Plato’s Protagoras. It goes through Cartesian skepticism to current time because it is quite reasonable for one to ponder in which way a person's inner reflections, mental states, etc. are connected with the actual reality.

We borrow the scheme of the argument from Hannon (2017):

(1) One cannot know that she is not in a skeptical scenario (SS).

(2) If one cannot know that not-SS, then one cannot know an ordinary proposition P, e.g. that one has hands.
(3) Thus, one cannot know that P.

However, we obviously have knowledge of many ordinary propositions including P. Here, we have arrived at a contradiction. Let us elaborate on the matter of applicability of the case. First, a skeptic does not try to convince us that we are in SS; his goal is to plant a seed of doubt that we cannot know that we are in not-SS. Thus, the skeptic is pushing us to commit an "epistemological suicide" by doubting our ability to know even P. However, the skeptic does not want us to fall into an epistemological paralysis. He wants us to hold some parts of our referential skills. He wants us to be able to perform some meaningful speech acts, e.g. meaningfully utter that we cannot know that P. The possible ways to deal productively with Skeptical hypothesis are to refute the premise (1) or refute (2). Let us compare the costs of these two alternatives.

First of all, let us state that the overall goal of epistemology is to provide a definition, or an account, for knowledge, i.e. a set of necessary and sufficient conditions for knowledge. This definition should propose a solution for both problematic areas in epistemology: Skeptical hypothesis and Gettier cases. Thus, if some approach plays well with a skeptic but fails in Gettier cases, then it is not good enough.

There are two approaches to build an account for knowledge: internalism and externalism. For the sake of simplicity, we will give only coarse-grained descriptions of those two ways. Internalism emphasizes the priority of internally fulfilled procedures for a person in order to have knowledge or, at least to have true belief. Simply put, if one knows that P, then one knows that she knows P (KK). This KK principle may appear to be too strong so we can weaken it by asking a knower to be merely aware of the base of her justification for P by using her inner reflections. In other words, a person should have reflective access to the base of her epistemological ground. Pure internalism immediately runs into trouble with Skeptical hypothesis: if our epistemological ground is accessible by mere inner reflection, then we cannot be sure that this reflection is linked properly with the external reality. The above premise (2) is an obstacle for internalism. One of our basic epistemic intuitions is to consider knowledge to be closed under logical implication. In other words, some sort of closure principle must be upheld. For the purpose of further analysis, we put here the closure principle (CP) in a very simplistic manner: (CP) if subject S knows that proposition P, and P entails proposition Q, then S knows that Q. Now, there is a big deal of subtleties. What does it mean that S knows P or Q? We can expect from S to have grounded knowledge, justified belief, or mere true belief. Should the epistemological status of P and Q coincide? It can be the case that epistemological ground for Q may be weaker than for P and we are still satisfied with such closure. Finally, the entailment also needs clarification because S's ability to make competent deductions is not a platitude. All those questions give birth to a set of ramifications of different CP. So, shall internalism refute CP? The cost of complete refuting is too high and internalists' accounts attempt to turn over the CP. Some modification of CP is needed, e.g. S knows that P and S does not know that not-SS but entailment that not-P cannot be made by S. We'll dig deeper later in the matter.

Externalist accounts add some conditions which are external for a person's cognitive or reflective abilities to the definition of knowledge. Those conditions are not in total control of the person. To name just a few examples, there are Pritchard's safety account, Greco's reliabilism, Goldman's relevant alternatives account, etc. (Pritchard, 2005; Reed, 2007). All pure externalist's accounts run into problems with Gettier-like cases unable to eliminate all possible environmental luck. Using Kripke's semantic of nearby possible worlds, if we demand from a person to apply a reliable method or a safe method of obtaining knowledge, we can be forced to admit that the person gains knowledge by gazing into a magic ball because by sheer luck the person's beliefs could be safe and/or reliable. However, those accounts perform well in the Skeptical scenario refuting the above premise (1). First, the SS-world is not a nearby possible world to our actual world. Second, in any nearby possible world to SS-world, a person can easily come to a false proposition by volition of the force governing the SS-world. Thus, a person's knowledge is neither safe nor reliable. Still, some troubles with CP occur for externalists as well. Though S knows that not-SS, she knows it on the basis of non-reflective procedure, i.e. contradiction to a particular external condition. Yet, externalists would experience difficulties to disagree that S knows that P, e.g. she has hands, by reflection. By any account of knowledge, we should be able to obtain some knowledge by reflection. Thus, by CP, one can infer from reflective knowledge of P that S is not in SS. How such a transition from reflective to non-reflective knowledge happened is a matter demanding an explanation. Thus, externalists are in need of CP modification also.

Having met all of the above obstacles, epistemologists turned to hybrid strategies for to define knowledge. We will consider Pritchard's path as an important example. His thinking went from a safety account (Prichard, 2005) through formulation of anti-luck virtue epistemology and ended up with a modification of Wittgenstein's hinge epistemology (Prichard, 2016). Pritchard's solution for the Skeptical hypothesis is based on Wittgenstein's work "On certainty" (1969). Again, we omit subtleties of the account for the purpose to explicate the clear structure of the argument. Wittgenstein considered that in order to engage in any meaningful epistemic thinking, a person should start from a set of propositions which are not rationally revisable. He called them "hinge propositions" because if one wants to open an epistemological "door", the "door" must have hinges. Pritchard (2016) took his idea further. His proposal is that hinge propositions are not beliefs, at least not such beliefs that are meant by JTB account of knowledge. They are our basic commitments. Also, hinge propositions, or commitments, exhibit several key features.

First, they are not acquired by any specific rational process, e.g. via competent deductions; they are presupposed by the whole corpus of our knowledge. Second, they are not epistemically revisable. Unfortunately, the concept of hinge commitments is not exhaustively clear; therefore, there are objections which we will consider later. Yet, we can see some opaque areas of Pritchard's account right now. The trickiest feature is the acquisition process. How did we learn them? If we somehow extracted them from our knowledge and/ or epistemic attitude, then what ground did we have for these knowledge and attitude in the first place? Further, I will propose a concept of epistemic intuition as a tool for acquiring non-inferential commitment toward a set of basic propositions. 'But the problem with claims from the intuitive point of view is that they are not yet *philosophically respectable*' (Reed, 2007: 260; Reed's emphasis). Indeed, the purpose of this dissertation is to pay some philosophical respect to our intuition.

The second feature of Prichard's account also needs clarification. We are prone to revise our beliefs from time to time, even the very basic of them, if evidence pushes us to do so. Prichard's solution to this particular worry is elegant and solid. He notes that there are two types of commitments. One type is personal commitments. They are locally revisable. Locality here is meant to be a possibility of partial revision of one's commitments due to newly acquired evidence against some of the commitments. Thus, the proponents of the "flat earth theory" are not doomed completely. Yet, there is another type of commitments which Prichard called "über" hinge commitments. They are not personal; they cannot be revised locally. They are likely to exemplify the above mentioned "core epistemology". If one wants them to undergo a revision, it would be a universal revision of everything. The most interesting for our analysis is Prichard's solution for the Skeptical hypothesis: one needs only one über hinge commitment - one is not mistaken systematically about the world. For now, we note two implicit important characteristics of this proposal. There are not many über hinge commitments, probably just a few. The content of über hinge commitment is simple and easy to grasp, though it may not be obvious. Thus, in such a way, Prichard beats the second (2) premise of the Skeptical hypothesis: if one does not know that not-SS, then one might be systematically mistaken about the world and, therefore, one does not know that ordinary proposition P is true. This CP is not valid anymore because über hinge commitments cannot be a product of any rational process and even less the product of competent deduction.

Now we consider several objections to the concept of über hinge commitments posed by Zhang (2021). They are of particular interest for our study because Zhang appeals to cases from mathematics. Though some of Zhang's concerns seem to be reasonable, his overall critique is missing the target exactly because of the above-mentioned implicit assumptions about mathematical knowledge. He does not specify what branch of mathematics is meant in his cases and what sort of mathematical views are endorsed by an imaginary protagonist of the cases.

Reasonably, Zhang doubts the clarity of Wittgenstein-Prichard's account of an acquisition of hinge commitments because 'hinge commitments and ordinary beliefs are phenomenally similar' (Zhang, 2021: 3536). In this doubt, we can express solidarity with Zhang. Thus, let us elaborate on the topic of acquisition of mathematical hinge commitments, especially on the topic of what mathematical über hinge commitments may look like. Considering Arithmetic, for mathematical Intuitionists or Constructivist, über hinge commitments will contain a set of very basic propositions describing a transition from intuitive grasp of two distinct moments of time to the concepts of natural number 1 and natural number 2; this set goes together with Intuitionistic logic which excludes LEM. Thus, Intuitionistic arithmetic can be constructed. For mathematical Platonists or Fregean Logicists, the set of über hinge commitments in the field of Arithmetic is just Peano axioms. In both cases, we acquire them through the process of learning. Usually, the process starts from a figure of a teacher demonstrating in class, for example, two apples and asking the class full of kids something like: "Sam has an apple and John has an apple. How many apples will John have if Sam gives him the apple?" Thus, mathematical Intuitionism and Constructivism borrow their explanatory power and attractiveness from this setup. For Platonism and Logicism, the situation is more complex because they should explain how one can have cognitive access to the realm of abstract objects of natural numbers 1 and 2 which are extracted from the pair of apples and yet are not having causal relationship with them. One may argue in favor of different verbal descriptions such as mental grasp, cognitive ability to abstract, etc. We remember that if Platonism does have an obstacle, it is the access problem. What is important for now is the observation that acquisition of the über hinge commitments intuitively differs from more habitual rational processes, e.g. competent deductions. As we noticed, Zhang's questions about the acquisition are legitimate but they may be the responsibility of educational science or psychology. In epistemology, when we use the method of cases, we usually exemplify a person who has already obtained the commitments. Thus, the real distinction of commitment from an ordinary belief is located in the feature of non-revisability.

Zhang (2021: 3538) formulates a modified version of the Closure Principle allowing rational revision of previously acquired beliefs, e.g. accidentally acquired beliefs. Thus, one can revise in principle über hinge commitment through a rational process, therefore über hinge commitment is not different from an ordinary belief. In favor of this argument, Zhang puts forward a "math" case of testimonial knowledge: "Suppose that a mathematician came to believe in the truth of theory T on the basis of his colleague's testimony. Later, the mathematician independently proved that T is true".

Zhang notes that mathematical truths are necessary truths; nevertheless, they are revisable. Über hinge commitments seem to play the role of necessities, therefore they are revisable. Here may be the case of faulty logic. All über hinge commitments are necessary but not all necessary truths are über hinge commitments. Yet, let us take a closer look at the "math" case because it's important not only for a particular epistemological topic but is also crucial from a methodological point of view. Let us ask who this mathematician is. If he is either a mathematical Intuitionist or a Constructivist, then he will just laugh because he would not believe T on the testimonial basis; he would not believe even the disjunction T or not-T until explicit constructive proof is present. A Platonist or a Logicist could, in principle, obtain a sort of weak belief in T and even experience an obligation to revise T. Nonetheless, it would be rather a case of a more or less complex theory T which is an unlikely candidate for being "über". He would try to obtain independent proof of T by going back to a set of axioms A which are essential for T. His purpose would not be a revision of A but a chain of deductions starting from A and reaching T. Probably, the "math" case should be reformulated as the following: a mathematical Platonist came to believe in the truth of an axiom on the basis of a testimony of his colleague and later revised this axiom. This description is much better and it entails only two interpretations. First interpretation is connected with the substitution of one axiom by another in the set of axioms of a well-established branch of mathematics. For example, consider Euclidean geometry on a plane - it has a set of axioms. Now, if we substitute the Parallel postulate with some other axiom, we obtain a ramification of hyperbolic geometry. We cannot argue that Euclidean geometry is better than that of Lobachevsky. They are both consistent in themselves. This gives birth to mathematical pluralism or, at least, its weak form: 'every consistent mathematical theory consists of truths about its own domain of individuals and relations’ (Zalta, 2024: 1). This substitution is not a revision of the previously existing set of axioms; it is a discovery of a new set that, nevertheless, contains some axioms which were in use in the previous set. Thus, the mathematician did not revise Euclidean geometry, the set of axioms of which is consistent, complete, and has the most possible minimum number of axioms - thus playing the role of über hinge commitments for Euclidean geometry. The mathematician discovered a new area of geometry and is, possibly, in the position of the necessity to check the new set of axioms for its consistency and completeness in order to be a good candidate for über hinge commitment for the hyperbolic geometry. Now, one may wonder whether one grand mathematical theory underlying and explaining all the others exists. The Set Theory is a probable candidate for this. Yet, this project is too far from its fulfillment; its perspectives seem to be even smaller than the attempts of physicists to provide a unified field theory. At the very least, for a while, theoretical - but not applied - mathematicians will dwell in the Platonian-like realm of mathematical pluralism where a metaphysical competition between the concepts of “natural number” and “geometrical point” is meaningless. Surely, an applied mathematician can see different merits in the application of different theories. If one measures a small plot of land, she would do better to use Euclidean geometry. If one travels around the globe, her navigation system should apply hyperbolic geometry.

The second interpretation of the “axiom” case is connected with the procedure of making a choice of axioms, or basic principles, which aims to squeeze the existing set into the most possible simple form. It is likely to happen in the field where the mathematical work is in progress regardless of how one can describe the work - be it discovery or invention. We already saw some tricky problems with the Set Theory in this regard. Yet, if it happens that a particular axiom could be derived from a narrower set of its “comrades”, then it entails that we were wrong about the “über” status of this axiom. It was a personal hinge commitment of some mathematicians including the person who proposed the non-optimal set in the first place. The work related to proper formulations of über hinge commitments in all possible areas of our epistemological concerns is not finished. For example, Prichard's formulation of über hinge commitment - one is not mistaken systematically about the world - can be not optimal; so, one can try to make a better one.

However, Zhang (2021) feels that empty arguments about mathematical cases are not convincing. If one proposes a “math” case, it is better to do some math. Thus, Zhang argues that ‘“2+2=4” arguably functions as a hinge proposition within our mathematical system’ (2021: 3543). Yet, it is provable and, therefore, revisable. Consequently, it is not a hinge proposition. I agree fully; it is not a hinge proposition and it never has been. It may serve as a personal hinge proposition for an unqualified person. However, any mathematician would immediately ask which “mathematical system” is meant in this example. The proposition “2+2=4” can be false in Modular arithmetic, for example. Thus, it is reasonable to suppose that Zhang considers our “usual” arithmetic, e.g. Peano arithmetic. Indeed, it is true in Peano arithmetic and it has to be proved despite its simplicity. Yet, no qualified mathematician would use the above proposition as an über hinge commitment. A good set of candidates for such usage is already established: a set of Peano axioms. No qualified mathematician would revise Peano axioms wondering whether they may serve as a basis for Peano arithmetic. It is established that the set is consistent, complete and minimal, if we restrict our consideration to first-order theories and reformulate the axiom of induction as a first-order scheme. We are in a lucky position; while Zhang is under an obligation to establish that no proposition can be used as über hinge commitment in Peano arithmetic, we can demonstrate only one such proposition in order to defeat his argument. Thus, let us take the first of Peano axioms: “zero is a natural number”. It is a proposition. It is true. One cannot have deductive justification for this proposition. The truthfulness of this axiom is acquired via careful inspection of its contents and understanding of concepts “zero” and “natural numbers”. Our justification is not inferential, we do not infer the truth value of the axiom from any other more basic propositions. We just grasp it. For the sake of simplicity, I will omit here the complex matter of relations between standard arithmetic and the constructive one (Plisko, 2022).

It is important to reiterate that our goal here is not vicious defense of Prichard’s account. I may be well dissatisfied with a particular formulation of a particular hinge proposition as Zhang does. My first goal here is to give enough reasons to the reader to see that even in the most stable areas of our knowledge, we are likely to use some such propositions that their truth values are established via non-inferential, probably intuitive, procedures. My second goal here is to appeal to the cautious usage of mathematical cases. For example, Zhang seems to uphold Platonism, at least implicitly. We'll see later that he notes a Platonist-like access to mathematical knowledge in his other case. Yet, Wittgenstein’s mathematical views drifted during his life and work from Logicism to mathematical Intuitionism. The question whether it means that Wittgenstein's hinge epistemology presupposes the former or the latter position is not considered in detail neither by Prichard nor by Zhang.

As we have noticed, the method of cases entails the need to provide a case in order to defeat some account of knowledge. A mere exercise in theorizing is not good enough. Thus, for example, Zhang must present a description of a situation where a person should doubt Prichard's über hinge commitment, namely, that the person is not systematically mistaken about the world. Zhang (2021: 3545) introduces a case called “chaos”. The description goes as follows: imagine Sam who grew up in the world subjectively identical to the normal actual world. Unfortunately, a day “X” comes and Sam begins to experience a world of chaos from time to time. All perceptions of all objects are uncertain and flexible. ‘Eventually, not a single physical object that Sam could discern remains’ (Zhang, 2021: 3545). Further, Zhang argues that because Sam has formed his referential system in the world subjectively (internally) identical to ours, he can hold the meaning of the words “world”, “being mistaken”, etc. Therefore, Sam can meaningfully exclaim: “I have been systematically mistaken about the world”.

Zhang’s case will not withstand methodological objections; it violates the above-mentioned intuition of extension. Zhang seems to jump to conclusion by asserting that Sam has the same reasons to uphold, at least partially, his system of references that gives a basis for meaningful speech as, for example, a person in the “brain-in-a-vat” scenario. It is wrong because the person in the “brain-in-a-vat” scenario is exposed to a faulty but consistent experience and, therefore, forms and holds her system of references. Sam’s dilemma is deeper. Sam’s chain of thoughts could look like the following: “If I have been systematically mistaken about the world, then my system of references should be abandoned as a whole and I cannot utter any meaningful sentence. I am epistemically paralyzed. Yet, I still think. Thus, my system of references is working. I formed it before the day "X", therefore, something went wrong on this day. The problem can be with me or with the external world. In any case it would be unfair to say that I have been systematically mistaken about the world. I can only assert that I have been mistaken about the world since the day "X"". Zhang tells us that in the chaotic world, Sam still 'retains access to Platonian mathematical truths' (2021: 3549). So, Sam is a mathematical Platonist. If he was an Intuitionist or a Constructivist, he would be in a position of holding the knowledge of the flow of time even in a chaotic world by the means of intuitionistic ground of mathematical knowledge; his bewilderment about the external world would not be total. But being a Platonist puts Sam in trouble. Yet, he would certainly consider the question of, for example, the Quine-Putnam indispensability argument first, before committing epistemic suicide. Even a possibility of considering such arguments as Quine-Putnam's prevents Sam's epistemic suicide. Thus, the day "X" is hindering the flow of Zhang's case. He decides to strengthen it by putting Sam into a "brain-in-a-vat" scenario where Sam is exposed to chaotic experiences from the day of his birth. Zhang thinks that in such circumstances 'although Sam's language may be quite different from ours, his statement, "I have been systematically mistaken about the world," would remain essentially unaffected' (2021: 3548). Surely, it would be affected crucially. If Sam was exposed to real chaos, he would not be able to form any meaningful language because in this chaotic world, no object had stability. If Sam has formed a language, then his "brain-in-a-vat" world is just drastically different from ours. But it is not a Skeptical hypothesis any more. Skepticism attacks our connection with the actual world or a world that looks actual. Epistemology is not concerned with people's experience of unstructured and non-logical referential systems. It is rather a matter of other humanitarian sciences.

Yet, there is a very important observation in Zhang's arguments. He notes that the real and probably unavoidable problem with Skepticism is the doubt about 'existence and predictability of the *external world*' (Zhang, 2021: 3549; Zhang's emphasis). To the best of my knowledge, this emphasis has not been yet highlighted in detail in epistemological studies. Thus, let us focus on this crucial matter.

When an epistemologist uses the term “world” when considering any epistemological conundrum, epistemological position in relation to the question of what is real should be stated explicitly. As we have seen, some philosophers, e.g. Platonists, Logicists, can consider two realities: the reality of concrete objects, or reality of physical, external world, and the reality of the realm of abstract objects, e.g. mathematical objects. Of course, they are obligated to make sense of how those two realities interact if they do. For such philosophers, the Skeptical hypothesis, for example, may be the question about all the possible knowledge including concrete and abstract objects. Thus, like presumably Zhang, they can come to believe that our knowledge of concrete objects, or the external world, is under threat, yet our knowledge of abstract objects is unaffected by Skepticism. Even among this group of philosophers a disagreement can exist regarding the degree of satisfaction related to the obtained answers.

One may be satisfied with stating that our knowledge of the external world is not completely safe while not all of our knowledge is unsafe. Another may be convinced that this result is unsatisfactory. However, there may be a group of philosophers who uphold the position that abstract mathematical objects don't exist. They are somewhat extracted from the external world. This is a spectrum of philosophers: from mathematical Intuitionists to radical social Constructivists. For them, the questions about our knowledge about the world and knowledge about the external world are essentially the same, therefore, an answer to a skeptic should be the same for both alternatives. It seems hopeless to expect that both groups can sit peacefully at one table. The only exception is if we take, for example, a proponent of mathematical Intuitionism and present to him a case where the Creating Subject operates on a time scale at will. But as we saw in the above poetic metaphor in mathematics, it is most likely that the proponent of mathematical Intuitionism will exclaim: “It is not epistemology, it is theology”. Despite my personal respect for human theological endeavors, we are not doing theology here.

To clarify the above tacit distinction, we will consider in more detail Reed’s (2007) analysis of epistemic fallibilism vs. attributabilism. Though I disagree that Reed’s arguments present us with a new sort of Skeptical hypothesis, I certainly believe that Reed highlights a new facet of the problem. Reed is discerning two roads to skepticism. One is a shortcut presented by the above Skeptical hypothesis. The other is a long road leading to the unavoidable gap between epistemological fallibilism and attributabilism. Fallibilism is a claim that if S knows that P via justification J, then it could be the case that S's belief in P might be false or accidentally true via J. Fallibilism is a warning about contingency that seems to be inseparable from our world; a lot of our knowledge, if not all, is only probable. Attributabilism is a claim that looks 'like a truism - if S knows that p, then of course the knowledge of p is attributable to S' (Reed, 2007: 238). Yet, epistemological attributabilism is a claim stronger than just a statement about the usage of a subject S as a mere locus, where a set of external/internal events has collapsed resulting in S's belief that P. Attributabilism presupposes an epistemic intuition that S's beliefs entail S's motivational component: S's will, desire, etc. In some sense, S is responsible for her beliefs, like one can be held accountable in a court for her actions even if the causal chain of those actions can include features beyond S's control. Thus, total attributabilism entails the subject's infallibility: no mitigating circumstances in the court. If a driver causes an accident because of his poor judgment of weather conditions, he is held fully responsible; he should know better. As Reed notes, a substantial degree of attributabilism is supported by almost all of the contemporary renowned epistemologists: Prichard, Sosa, Greco, Neta, etc. Yet, the whole castle of modern science is built on fallibilism, therefore, there is an unbridgeable gap in our knowledge that comes in a degree: more fallibilism, less attributabilism, and vice versa. A clarification is needed here. This gap is not a matter of only lucky evidence, Prichard calls it veritic luck, but also a matter of reflective luck. Reflective luck can be illustrated by the subject's reflection that, indeed, über hinge commitment is true and I am not systematically mistaken about the world; yet I am lucky to dwell in such a benign universe. It resembles, for example, the Quine - Putnam indispensability argument. Thus, Prichard's solution stands, but a tiny bit of anxiety is unavoidable. Pritchard (2016) calls it epistemic angst.

Finally, we can point out an obligation for epistemologists to draw a line of how the existence of abstract objects is connected with our world. A Platonist and a Fregean Logicist can say that while knowledge of the external world is fallible and, therefore, only partially attributable to S, S's mathematical knowledge is infallible and totally attributable to S. For such epistemologists, the question of explaining our access to the realm of abstract objects is difficult. A mathematical Intuitionist and a Constructivist can say that all knowledge is fallible and, therefore, only partially attributable to S. Those epistemologists run into trouble with explanations of how we seem to retain mathematical knowledge in almost any possible and bizarre cases. Moreover, a very important observation shall be made clear: the above epistemologists should experience much less anxiety related to mathematics compared with the external world. In any case, mathematical knowledge is more stable. It is stable as long as the subject S retains a bit of her system of references, i.e. as long as S is sane and not epistemically paralyzed.

Now we turn to the analysis of problems with Skepticism in Moral epistemology. As it happens, the above problem of dichotomy of epistemic status of abstract vs. concrete realms is pivotal for defeating the strategies of Moral Skepticism. It is noted that moral propositional knowledge, if such exists, looks like a proposition of the form “F is M”. Here, “F” is some sort of fact from our real world; it can be an observation or a description of a phenomenon. “M” is a moral predicate asserting that F is moral/immoral, permissible/impermissible, good/evil, etc. Thus, we can see that, in some sense, Moral epistemology is a bridge between two realms: abstract and concrete. Consequently, this is possibly the most difficult, and yet most important, project for epistemological anti Skepticism: to present a good project for the bridge.

However, Moral Skepticism comes in many forms. One can argue in favor of epistemic Moral Skepticism by challenging moral knowledge, or justified moral beliefs, or even simply truth conditions for moral beliefs. One can support moral agnosticism by arguing that neither moral knowledge nor its absence has solid justification. Another skeptical path is to refute moral realism which is a premise about the reality of moral entities. Finally, there is practical skepticism: a view that irrespectively of a set of one’s moral beliefs, one can never be sure how to behave morally. The major arguments in support of Moral Skepticism are diverse too. There are arguments from moral disagreement, best explanation, moral regression, and Skeptical hypothesis. For our analysis, it is important that all of these arguments are mutually supportive. Thus, our purpose is rather to defeat the above set of skeptical arguments. We will not dig too deep into the subtleties of the Moral Skepticism per se. We will take moral nihilism as a "guinea pig" and consider its supportive base step by step.

One may argue in favor of moral nihilism, i.e. nothing is morally wrong (Joyce, 2001). Basically, it is an analogue of Skeptical hypothesis in Moral epistemology. Note that moral nihilism is not bound by the general form of epistemological skepticism. For example, one can feel perfectly fine endorsing an über hinge commitment as a defeater for Cartesian skepticism, yet she may claim that nothing is morally wrong. The possible line to deal with the problem is to propose something akin to hinge commitment in the realm of morality. A skeptic is likely to appeal to the deep disagreement regarding moral issues demanding to present a particular proposition as a candidate to be "über"; otherwise, we can fall into moral relativism. Let us illustrate the idea through an example of an even more controversial phenomenon than moral beliefs - religious beliefs.

The main idea is that if a hinge commitment does not have to undergo rational evaluation while avoiding the status of irrationality and forming a base for our everyday beliefs, then why shouldn't we bestow the same tolerance on our religious beliefs? Prichard calls this position "quasi-fideism". The major benefit of quasi-fideism is that we can cease to paternalistically consider our religious fellows as mistaken irrational subjects. Yet, the most severe objections against hinge epistemology as a whole are raised over its religious consequences (Salvatore, 2019). Because of the "locality" of the Wittgensteinian rational evaluation of our beliefs, we run into relativism. As Salvatore put it: 'if our epistemic practices are all based on a-rational commitments, then every epistemic community could legitimately hold its own practices, as they all rest on commitments that are both unquestionable and a-rational, thus outside any form of epistemic evaluation' (Salvatore, 2019: 71). As we saw in the case of the external world where abstract entities are not involved, Prichard seems to propose a convincing response to epistemological relativism: a quest for "core epistemology". In the case of religion, it is 'impossible to solve disagreement between epistemic agents with radically different worldviews' (Salvatore, 2019: 71). We do not learn religious commitments by gut. The whole corpus of theology can be seen as an attempt to negotiate about basic religious “truths” or commitments. Thus, there can be local communities that hold incommensurable, non-negotiable religious hinge commitments: ‘our agents are in a sense “epistemic pairs”; both their belief system and practices rest on a number of different, but equally a-rational, hinge commitments’. (Salvatore, 2019: 73). However, it should be noted that the real issue raised by Salvatore is not relativism but sectarianism. To be a real objection for quasi-fideism, different belief systems must have nothing in common. They should be even epistemologically closed in their referential systems: one cannot know our God if she is not practicing our ways. Do we have an analogous situation in morality? We don't because it is hard to imagine two different local communities that can run into moral sectarianism in relation to each other's moral beliefs. Even the most distant moral groups, e.g. religious fundamentalist and modern globalists, can find some common moral ground. Who will disagree that it is wrong to torture a child seeking sexual pleasure? Thus, for moral skeptics, the disagreement argument unfolds as a skeptical demand of total moral agreement among humans. It is obviously too much to demand.

The next move for a skeptic is to appeal simultaneously to the “best explanation” criteria and to the regression argument: if S knows some moral proposition, then S knows it through a chain of inferential cognitions or in a non-inferential manner. Therefore, somewhere in the S’s reasoning has to be a proposition known non-inferentially. However, if there is such a proposition as M, the skeptic will be able to provide a better explanation of why M is true, e.g. via psychological, sociological, anthropological, evolutionary, etc. reasons. We agree with the skeptical demand that we do need to provide an account of non-inferential moral knowledge. Yet, another skeptical demand is too strong: it might be the case that some explanation E seems to be “better” than others. E can look simple by involving less variables; variables can look better defined through empirical practice. However, if E entails some unacceptable moral attitude, then it would be better to sacrifice the elegance of E. Thus, one may say that there is knowledge of ethical propositions but these propositions are not moral. Let us say that they are natural, e.g. they follow from social and economic explanations. For example, slavery is bad but it is bad because it ceased to be socially and economically effective. Therefore, we can argue that there is nothing wrong with slavery per se. There were times in the past when slavery was used effectively. If it happens that our social tapestry changes significantly, we could consider buying and selling people again. This example shows that an obligation for an anti-skeptic is to provide a plausible explanation but not necessarily the "best" one; it may happen that a plausible explanation without undesirable consequences is the best.

One of the promising responses for the above skeptical attacks in Moral epistemology can be the Moral Intuitionism approach. There are two major approaches to expressing the understanding of what basic moral knowledge is: "self-evident" propositions (Audi, 2008) and intellectual "seemings" (Huemer, 2005). Audi's account elaborates on the matter of what "self-evident" propositions are: non-inferential propositions understood as propositions the truth of which can be known on the basis of careful inspection of their conceptual content. This account is close to Ross (1930). Noteworthy, Ross defined them as convictions made by well-educated people and based on ethical data. Though Audi and Ross are Intuitionists, they mostly avoid precise analysis of the intuitive machinery at work. Huemer (2005) emphasizes a particular way of how we should consider the work of intuition and, as consequence, how the basic blocks of moral knowledge look. For Huemer, our moral intuition works in a way similar to our perceptions: some object is red because it seems red. Such ramifications of Intuitionism can be called quasi-perceptual. Moral truths are apprehended and/or grasped immediately and non-inferentially by our intuition; they are intellectual "seemings". Huemer's account can tackle a subtle matter of whether the basic moral propositions cannot be proved or need not to be proved. Intellectual "seemings" cannot be proved and, surely, they need not. Thus, we avoid the shaky business of speculation about what it means for a proposition to have a "self-evident" justification. However, an objection related to a disagreement is quite serious for Huemer's proposal. The intuition per se renders an unreliable instrument. Andow (2018) has shown that moral intuitions are susceptible to framing, i.e. one can report different intuitive judgements if a different order of description of a moral conundrum is presented to a person. Also, the set of moral intuitions seems to be very volatile and diverse even in relation to such seminal problems as the Trolley problem.

Now we turn to the analysis of moral propositions as “self-evident” truths understood in the spirit of Ross’s and Audi’s works. We will not wonder which is better - the Audi's (2008) “containment theory” or the concept of “moral fixed points” (Cuneo and Shafer-Landau, 2014) - as these epistemic subtitles are rather about a particular machinery of one’s referential system. We are interested in dealing with broader philosophical speculations.

The first major objection stems from the Open Question Argument that roughly says that if claims of a kind “X is N, then X is M” where N is natural predicate and M is moral one are all open questions, then such claims cannot be conceptual truths. As Lutz (2015) explains, an open question is ‘one whose answer can intelligibly be doubted by conceptually competent individuals after reflection on the concepts involved’ (2015: 49). Thus, one can argue in favor of the Open Question Argument objection referring to historical facts, e.g. ‘the ancient Romans, with their love of gladiatorial blood sport, clearly did not think that recreational slaughter was always wrong’ (Lutz, 2015: 50). We cannot call ancient philosophers morally incompetent; therefore, the proposition “recreational slaughter is wrong” does not express a conceptual truth, though it may seem right and obvious for many today.

First, we certainly can call ancient philosophers morally incompetent, at least in some cases. Why not? Lutz seems to use an unconscious premise that afflicts many moral arguments: there were and there are a lot of people who possess a lot of moral knowledge. What if this premise is wrong? Why is anyone entitled to count herself a righteous person? It may happen that righteousness and vice are both exempt from presumption of innocence and shall be proven by a person who attempts to cast a moral statement.

Second, the Open Question argument is valid, yet it turns to support the concept of self-evident propositions. In the spirit of Moorean argumentation, we can assert that if predicate M cannot be explained in terms of natural predicates such as N, then M is non-natural - it is abstract. The question about the necessary condition of “being closed” for an abstract entity in order to be conceptual is far from clarity. We saw above that it is still permissible for a qualified mathematician to ponder on the matter of what a “set” is without undermining the conceptual essence of the Set theory.

The second substantial group of objections to the account of non-natural moral “self-evident” propositions as conceptual truths attacks the thinker and her way of thinking rather than the content of the above propositions. Following Audi (2008), a concept of vixen is “contained” in the concept of female; therefore, it should not be possible to think about the former without thinking about the latter. Yet, it is possible to think about “killing” without “wrong”. Thus, the theory of “conceptual containment” does not stand. This line of attack is also supported by the demand to feel obviousness as ‘something's seeming obvious after reflection is still a hallmark of conceptual truths’ (Lutz, 2015: 54). Here, we can also see several somewhat worrying presupposed premises. Conceptual truth is likely to be aesthetically simple, yet it can be not obvious, at least for a substantial period of time until one’s referential system is fine-tuned through moral reflection. Indeed, one can think about the word “killing” without the word “wrong”. Can one think about the concept of “recreational killing of a human” without an intuitive hint that something is off? Another thought is that a hunter can think about recreational killing of animals without awkwardness. Does it make hunting morally indifferent in all circumstances?

We may call the third group of objections to the concept of self-evident, non-natural, non-inferential propositions aesthetic discomfort regarding the justification method: it is strange to say that “recreational killing is wrong” on the basis of understanding that it is wrong. Again, here we find the stubborn assurance that moral competence is an easy activity. For example, let us seriously ponder on the essence of the property of “being a human”. Since the beginning of human civilization, we have tried to understand the concept. Did we understand the concept? If I am honest, I am obliged to say that I do not, not entirely.

To finalize the depth of the concept of self-evident non-natural moral knowledge, it is important to note Ross's strain of thoughts related to normative character of moral knowledge. Ross thought that self-evident propositions are rather assessments of something that counts pro or contra of some act; they are prima facie duties. Thus, what we ought to do is a complex process of decision-making, weighing all elements involved; it is unlikely that a set of self-evident moral truths will give us clear instructions for any occasion. Again, one can be disappointed that an Intuitionistic account does not provide an easy algorithm to navigate the world. Yet, Moral epistemology is an unlikely candidate to fetch psychological comfort.

Still, one may easily feel that all the above equilibristic with the moral institutions just pulls us away from a simple natural explanation, e.g. evolutional, psychological sociological, etc. In the next chapter, we will build an analogy of knowledge of basic moral propositions with the knowledge of mathematical propositions. We will use very simple mathematical theories as our illustrations. We hope that this approach is tangible enough, at least for the purpose of shaking moral nihilism.

3. Methodology. Analogy as Philosophical method

In the previous chapter, we explored methodological approaches that serve as a foundation for the entire project of Analytical philosophy. This exploration was important to avoid oversimplifying the process of philosophical inquiry. Such inquiry is not a mere process of refining a set of deductions; it always involves one's metaphysical attitude and corresponding intuitions, even for those who would like to dwell solely in the realm of deductive statements.

Now we will concentrate on our specific methodological tool: building an analogy between moral knowledge and mathematical knowledge. It's essential to clarify what sort of analogy will be used. In philosophy, analogy, or analogical argument, is not a figure of speech. It has its own domain of philosophical theories. Analogical reasoning is deeply rooted in human reasoning; it is used in basic learning activities and complex epistemological processes. For example, consider the Poincare hypothesis. While it seems straightforward for 2-dimensional surfaces, its 3-dimensional "comrade" is unlikely to be understood without analogical reasoning. In the broadest sense, an analogy is a comparison of two systems of entities that highlights the similarities and differences between those systems. To put it in quasi-formal terms, we suppose that there are two sets of entities S and T: S is called the source domain and T is the target domain. S is similar to T in some known respects and S has a feature of our interest Q. Therefore, T has a feature Q* which is similar to Q. All arguments of such sort are ampliative, i.e. their conclusion does not necessarily follow from their premises. The conclusion always has a tentative character.

There have been several attempts to strengthen the argument in order to demonstrate that its conclusion can be shown as guaranteed to a higher degree, such as Russell's determination rule (Russell, 1986) or Norton's material theory (Bartha, 2020). These attempts are characterized by providing a more formal structure for the argument and extracting particular sets of deductions and inductions. However, they eventually turned out to be excessively restrictive, making well-formed and intuitively reliable analogical arguments seem unreliable.

There is another group of approaches that is rather too inclusive: ‘quasi formal theories of analogy’ (Bartha, 2020: 3). Bartha (2020) refers to his theory of articulation model and Hesse’s (1966) theory. Those theories emphasize the importance of horizontal and vertical coherence in the source and target domains. According to Hesse’s (1966) criteria, good analogies have the following characteristics. First, there are horizontal similarities in observable properties between entities of source and target domains. Second, there are similar vertical causal relations in both domains. Third, there is no essential difference between domains, i.e. essential properties and causal connections in the source domain and corresponding properties and connections in the target domain are not significantly conflicting.

We will employ this set of quasi formal criteria for ethical-mathematical analogy. Bartha’s (2020) proposal seems even less demanding. Yet, finding a middle ground is reasonable since a complete and consistent theory of analogical reasoning is still under development.

We also need to discuss the justificatory power of analogical arguments. Conclusions of analogical arguments are not guaranteed. We can understand them in two ways: probabilistic and modal. The probabilistic approach allows us to make statements about the degree of plausibility of a hypothesis that a given feature is present in both domains. However, such estimations are very likely to demand a formal approach to the arguments. The modal approach seems to be consistent with the above quasi formal theories: ‘we need to show that the same type of connection that holds in the source domain *might* hold in the target’ (Bartha, 2020: 8; Bartha's emphasis). Thus, our justification does not try to establish more than prima facie plausibility.

Finally, we consider one more important variability in analogous reasoning. Analogous arguments can be seen as demonstrative, explanatory, predictive, paradigmal, etc. We will use explanatory arguments for our ethical-mathematical analogy of the following form: ‘Q explains P in the source domain. Q* (if true) would explain P* (analogue for P) in the target domain. Hence: Q* is plausible’ (Bartha, 2020: 7). This particular methodological approach aims to engage modest moral skeptics or agnostics and propose a plausible and convincing alternative to the claim that there is no non-natural moral knowledge.

As a source domain for analogical argument, we will use a “toy” model of mathematics consisting of two theories. One is Euclidean geometry on a plane, the other is Hyperbolic geometry. Each contains a consistent set of true propositions which are conceptual, or non-inferential, i.e. set of corresponding axioms. Those propositions concern abstract, non-natural objects and relations among them. I doubt that any qualified mathematician would be willing to put the epistemic value of those axioms under scrutiny.

There’s debate about the potential existence of the underlying mathematical theory explaining all mathematics, such as the Set theory, although this is beyond the scope of this dissertation. Our goal is modest: while we trust mathematical theories to a high degree, we lack similar trust in theories of Moral epistemology. Thus, we aim to push some moral theories toward gaining epistemic trust and value. It is important to note that we will not develop a Prichard's account of über hinge commitments in mathematics or Moral epistemology. As noted, mathematical axioms of a well-established branch of mathematics are unlikely to be rationally revisable. These axioms can provide better epistemic anchoring than the concept of commitment suggests; this depends on one's epistemological attitude toward the philosophy of mathematics. If one views a realm of abstract mathematical objects and a realm of concrete objects as two separate entities, then one is unlikely to experience Prichard's epistemic angst in mathematics. There is no unbridgeable gap between mathematical fallibilism and attributalism. Therefore, one is not committed to the truth of axioms; one knows their truth.

In the above paragraph, we had a conversation with a mathematical Platonist. Our proposal to such a person sounds as the following: if you are a mathematical Platonist and, yet, you think that moral propositions are results of some interaction of natural factors, then you will be presented with a plausible explanation to reconsider the epistemic ground of moral propositions. However, it follows from the above analysis of the philosophy of mathematics that a Fregean Logicist and even a mathematical Intuitionist can join the conversation. They will need to do a bit of mathematics because some of them can doubt the non-inferential status of geometric axioms. Thus, they will need to go deeper into the number theory substituting geometric points and lines with their numerical coordinates. Still, the set of non-inferential propositions can be found by those mathematicians. Most likely, it would be Peano axioms. So, our choice of geometrical theories is rather a convenience for illustration purposes.

One more important clause is needed. As we understood, analogical quasi formal reasoning is some sort of structure-preserving mapping. Yet, it is not a construction of isomorphism and even less homomorphism. For the plausibility of the conclusion of analogy, the source domain does not need to be the whole body of mathematics. The source domain must exhibit clearly a set of key characteristics and features only. If we find those features to be present in the target domain, then we can be satisfied with the analogy. Also, we noted two geometries; both of them will be handy at the stage of the search for substantial disanalogy of our account of moral propositions. At this moment, the mathematical pluralism and its consequence for the ethical-mathematical analogy will be introduced into the discussion of possible objections.

To draw the line under the discussion of our methodology of analogy construction, we pose one subtle feature of our source domain. It occurred in earlier arguments as a misleading factor. For example, in Euclidean geometry, we can find axioms and very simple, almost obvious, theorems. For instance, one may dwell on the truths of the sum of angles of a Euclidean and hyperbolic triangles. At the very least, since one becomes familiar with the graphical proof that the sum of angles of a Euclidean triangle is 180 degrees, it is hard to undo the sense of one's certainty in it. The same applies to the negation of the analogous proposition related to hyperbolic triangles. Let us look at one more example. Consider the Parallel postulate - the conceptual truth for Euclidean geometry. Let's take two parallel lines. The statement that “if a third line is crossing one of the parallel lines, then it is crossing the second” is provable and, yet, it is an obvious theorem. We should not mix up deductively provable, yet obvious, statements with conceptual truths. Thus, in the further analogical mapping, we will probably find a similar distinction among moral propositions.

4. Main argument. A scheme to build an analogy between basic mathematical propositions and moral "self-evident" propositions

We have chosen Euclidean geometry as a source domain for ethical - mathematical analogy. Now we shall turn to the target domain, namely a description of some analogous moral theory which behaves similarly to the source domain. This moral theory need not to cover an extensive range of moral issues. By analogy, Euclidean geometry is not the whole mathematics; it is not even the whole body of geometry. Let us work on developing, for instance, a moral theory of "Killing".

We are going to work on the theory from scratch because our first premise is that we do not know a lot about morality yet. This statement may seem to be bold. Yet, it should be noted that this statement does not entail that we do not have moral intuitions or beliefs; it doesn't entail that we always behave immorally, etc. It simply doubts that we possess *knowledge* of a wide range of moral propositions. Let's propose an illustration which does not claim to be a strict description of historical processes but rather an example of possible development of our thinking. It is plausible that around ten thousand years ago, human society went through an agricultural revolution. Humans found themselves in a situation of needing measurement tools. Possibly, hunter-gatherers could successfully tackle the task of sharing the prey. Yet, fields and crop needed measuring. Through fulfilling new practical tasks, basic mathematical statements came to be discovered. At first, they were not axiomatic statements but rather a set of obvious theorems related to area of plots of land and count of crop. Gradually, a solid body of mathematical knowledge was developed. As Corcoran and Hamid (2014) put it, four forms of mathematical knowledge were developed: propositional, objectual, operational, and expert. Objectual knowledge of mathematical objects allows us to form true propositions via operational "know-how". Yet, there is the expert mathematical knowledge which 'is the knowledge possessed by an active qualified practitioner - the medical knowledge possessed by a physician or the mathematical knowledge possessed by a mathematician' (Corcoran and Hamid, 2014: 5). This picture shows that being an expert in mathematics entails usage of some sort of mathematical intuition. This intuition may come in the form of the above-mentioned concept of beauty and even internalization of an expert's personal fallibility. Now, what gives us ground to be sure that one has expert moral knowledge? Indeed, the question of moral deep disagreement raises doubts about morality. Yet, while one can doubt morality per se., it would be more moral to doubt one's expertise in morality, wouldn't it? I fail to see how we can try to refute the existence of mathematical knowledge per se. Everyone knows that “2+2=4”. A person who is not an expert in mathematics is likely to be sure that she knows what number “2” is as well. A mathematical expert would not be so sure if she were tasked with, for example, an explanation of the number “2”. Surely, while “2+2=4” is a platitude, the proposition “2 is a property of a set of all sets consisting of two objects” is not. Interpreting Bertrand Russell, Corcoran (2019) sees the activity of defining mathematical objects as activity performed by a mathematician in search for ‘deeper laws and deeper concepts by analyzing current mathematics’ (2). As we saw earlier, Dummett considers the task of formulation of applicability and identity criteria for basic mathematical objects to be the most difficult one. Nonetheless, there are no mathematical skeptics. If we suppose that ethical-mathematical analogy could be seriously considered, then the process of formation of moral knowledge has to be difficult. It would be a mistake to smuggle in our reasoning an unwarranted assumption that we are moral experts. We are not; we only strive to be them. The question of a figure of a competent moral speaker will be considered in later chapters. Now, we start from the very beginning by wondering what can play a role of moral intuition akin to the above mathematical expert’s sense of seeing the “beauty of a theory”.

We have decided to form an analogy between Euclidean geometry and a moral theory which we called “Killing”. What sort of intuition can help us assign a moral predicate to a case of killing? We propose an intuition of moral ugliness, or disgust. Probably the most reliable moral propositions in the theory of killing will be of the form “such killing is immoral”. Still, it would be moral knowledge and a refutation of moral Skepticism. Thus, an intuition of moral ugliness as opposed to mathematical beauty will play a role of the immorality detector analogous to detection of ugliness of mathematical theory, i.e. contradiction. Our proposal doesn't entail that disgust is the only intuition used in moral judgements. It just seems to have a good epistemic ground for our particular case. Disgust as a moral factor has empirical support: the correlation between disgust and severity of moral judgements is significant (Schnall et al., 2008). It is also theoretically grounded. My personal epistemological intuition informs me that people are better equipped to detect what is immoral. I can be wrong, yet throughout the whole history of humankind, there were systems of taboos. Those systems of taboos were formulated in a more precise manner compared with existing sets of rules of good behavior. Such well-known names in psychology as Sigmund Freud, Stephen Karpman, Eric Berne etc. give us a theoretical frame for the intuition of existence of an agency of moral assessment innate for a subject. One theory names it superego, another - a parent or rescuer. In addition, this agency is deemed to be in control, though sometimes in an unhealthy manner, of our more primal urges. Yet, I do not want to ground the intuition on psychology or sociology. I use it merely as an example. My goal is rather to show that, for example, if any person in our modern society is presented with a case of "recreational killing of a human", then she is likely to feel that something is off, something is not right. In our analogy we restrict the role of intuition, e.g. intuition of disgust, to be responsible for 'providing raw materials for thought, it does for the abstract realm something perception does for the concrete realm' (Audi, 2022: 68). For us, moral intuition is used as a flashlight when one is looking for a key on the floor of a dark room given that there can be multiple lost keys on the floor. After a key is found, it should be tested whether it matches the lock on the door leading out of the room. Such a cautious approach can garner good support from a recent article published by Audi (2022). In the sections dedicated to epistemological theorizing, a chain of well-formed arguments can be found and we will use them later. Unfortunately, Audi (2022) also attempts to provide an analysis of phenomenological tapestry of moral intuitions. After reading the corresponding sections of Audi's taxonomy, one sees that in morally laden situations, moral intuitions influence moral emotions and vice versa. Both moral features sometimes flow from and sometimes give birth to moral experience. The occurrence of those phenomena bears no plausible hypothesis of their correlations, much less causations. Audi's description renders a phenomenological mess. Let me explain by providing an example. In the section discussing promissory fidelity, Audi (2022: 63) proposes a case of a son sitting near the bed of a dying mother. The mother asks him to distribute some substantial sum of money from her wealth to one of her grandchildren - one of son’s nephews. However, this nephew is an alcoholic. And after the mother's death, her son ends up with a moral conundrum: to uphold his promise to his dying mother and give the sum to the nephew or postpone the transfer and use the money as a motivation for the nephew’s decision to undergo a rehabilitation program. Considering the case, Audi describes a broad range of the protagonist's mental phenomena such as guilt, regrets, hope, duty etc. Audi links them with the moral realm. It is likely a mistake. I shall invoke my personal expertise as I spent a good deal of time practicing as a therapist in the field of addiction. I obtained a proper professional education and training and even did some research (Korchevoi, 2022). As an expert, I can suppose that following the Karpman’s drama triangle, an alcoholic and his relatives are likely to play for a while a vicious cycle of interchangeable social roles: a victim, an aggressor, and a rescuer. They change those roles whenever they feel fit and use each other as a means, not as the end. A codependent relative of an alcoholic may have secondary psychological benefits of feeling righteous while enabling compulsive drinking by providing money or other resources. So, it is always a complex and psychologically unhealthy situation. Now I shall ask, does the dying mother in the above case know about the nephew’s addiction? If she does, then she deliberately asks her son to fulfill an immoral act of enabling an alcoholic. If she does not know, then she is mistaken about the moral value and the implications of her request. Both options put before the protagonist of the case a choice of what has more moral weight: a promise or an immoral request in the former situation, a promise or a morally false request in the latter. Thus, our protagonist is dragged into a situation where he will experience moral harm in any case; his feelings are justified. However, those feelings are rather a matter of a session with a good therapist. Thus, the answer to the moral conundrum is simple: if one dwells in the realm of sacred duties and promises to the dead, she will not have doubts about her actions. If one employs a more balanced approach avoiding objectification of herself and others, she will pay a price of psychological discomfort for doing the right thing - to postpone the transfer and put the troublemaker into rehab. This example shows that epistemologists should be cautious proposing moral recipes. It is possible that an epistemologist's duty is to explicate a scheme of moral knowledge, a way to get it right; the special content of moral knowledge and its expression can be a task for others. There can be many specific areas of demand for moral assessment; different expertise may be requested in different cases. There is a persistent assumption of existence of a big volume of easily disseminated moral knowledge. Even epistemologists tend to think that moral knowledge is possessed by many. I do not see why it is so. We are sure that knowledge is not a rare thing in the other epistemological areas. For example, everyone can have perceptual knowledge. However, moral knowledge can be a different beast. Returning to our account, we will use the intuition of moral ugliness, or disgust, as raw material. There can be more useful intuitions; yet, if by using one of them we can reach our target domain for analogy, then our goal is fulfilled.

Now we need to pick up a proposition that can serve as non - inferentially, or conceptually, justified abstract proposition. Let us borrow an instance from Lutz (2015): the proposition "recreational killing of another person" seems to be a good candidate. I fail to see how, in our modern society, one can assign a positive moral value to this proposition. It also seems to be simple, or even obvious; therefore, it is plausible to suppose that one can come to its knowledge by careful contemplation of its content.

Lutz (2015) does not see this proposition as conceptual truth appealing to two objections: disagreement objection and Open Question objection: 'the ancient Romans, with their love of gladiatorial blood sport, clearly did not think that recreational slaughter was always wrong' (50). Lutz develops his thought in such a way that we cannot call ancient philosophers morally incompetent; therefore, the proposition "recreational killing of another person is wrong" does not express a conceptual truth, though it may seem right and obvious for many today. Let us show that if the mathematical analogy were to be taken seriously, the historical examples would serve us quite differently. Consider a part of mathematical knowledge called Euclidean geometry. It is a useful theory with many applications. More than two thousand years after Euclid, Lobachevski and Riemann have built axioms of non-Euclidean, or hyperbolic, geometries, e.g. Elliptic geometry. In Elliptic geometry, or roughly speaking geometry on a surface of a sphere, one conceptual truth - namely Parallel postulate - is substituted by another axiom. However strange and theoretical these geometries may appear to a lay person at first, we definitely consider them useful since the Theory of general relativity supports the hypothesis that our space is non-Euclidian. It is worth to note that for a qualified mathematician, after a decent reflection, the decision whether to include the Parallel postulate or to substitute it with another can be a basic block of the system of axioms. Yet, one will hardly call this decision obvious considering the abundant and complex set of consequences. In addition, the above sentences expressing the Parallel postulate in Euclidean geometry and its substitution in Elliptic geometry can be rendered as "self-evident" because one needs only the right grasping of such concepts as a point, a line, a plane, and a sphere to assert their truth. One more crucial observation is that a mathematician who asserts the truth of the above propositions is not in any sort of contradiction. While dwelling on geometry, the mathematician switches her conceptual thinking about the space between plane and sphere. This conceptual shift is not a platitude or an obvious move, though it may seem simple after it is done. The shift in conceptual mathematical thinking will be useful in later analysis of mathematical vs. moral pluralism. At this moment, it is worth noting that it took more than two thousand years for mathematics to develop what seems to be the simplest branch of geometry, i.e. Euclidean geometry.

Now one can see that if we are going to use mathematical thinking as a template for moral knowledge, it is perfectly safe to suppose that some people or societies in the past have failed to recognize recreational killing as wrong. They could fail to contemplate on the concepts of human personality and its autonomy while trying to grasp the concepts of wrongness and killing. We do not expect from Euclid to know all possible ramifications of geometries, do we? We do not even expect Euclid to come up with an exhaustive set of all possible theorems of Euclidean geometry. Let us consider the moral theory of killing and our understanding of the concept of autonomy of a human. We can suppose that in comparison with ancient philosophers, we have developed either the concept itself or our understanding of it. The former is a position of a moral naturalist and the latter is the opposite because the non-natural concept can exist untouched by a human mind. Thus, even a hypothetical Ancient Roman could say that "killing of an aggressor in self-defense is right" analogously to the Parallel postulate. Yet, he could fail to consider the wrongness of "killing another human for the purpose of entertainment” analogously to Riemann's geometry because he was not morally proficient enough to contemplate on slavery, humanness, etc.

Yet, in our speculation related to recreational killing, we have involved such terms as “person”, “autonomy”, “humanness”. Lutz (2015) can be right that the wrongness of recreational killing is not a conceptual truth but rather an inferential one. It does not entail that there are no conceptual truths. It entails that our candidate for being a conceptual truth akin an axiom is simply a moral theorem following from another more basic proposition. Thus, when employing the above moral intuition, we should ask what makes recreational killing disgusting. It is plausible to suppose that the moral ugliness of recreational killing is failure to conceive another human as a human, in objectification of a person and taking pleasure in this objectification. Thus, our next candidate for being an analog of one of the axioms can be the proposition “objectification of one person by another person is immoral”. Let us make an important observation. We are dwelling on a moral theory which we call “Killing”. There can be other theories, for example “Sport”. Surely, it is easy to see that at a boxing match, two opponents are objectifying their bodies, hurting each other. Of course, they do it deliberately. Simultaneously, they follow the boxing rules, they preserve their humanness, at least to a certain degree. It is unclear to what degree one’s body is immersed into one’s personality. Yet, we may see that the theory of “Killing” and the theory of “Sport” are different in one critical outcome: no one is expected to fight to death while boxing. One may see here a mere contextual distinction. I propose to consider those two theories as conceptually different. A mathematician can grasp the concepts of a point and a line thinking in different conceptual sub realms of geometry: Euclidean and non-Euclidean. In both geometries, the concepts of geometrical “points” and “lines” will behave very similarly; yet, immediately after a conceptual thinking shifts from a flat surface to a curvature, the behavior of parallel lines in Euclidean space and their analogs in non-Euclidean shifts as well. Thus, the concept of objectification used in our theory of “Killing” is always a concept of aiming to terminate someone’s life. In addition, keeping in mind that the proposition “objectification of a person by another person is immoral” can be considered only one of the “moral” axioms, a set of “moral” axioms is needed to cover the theory of “Killing”. For example, we will need propositions related to the actors' premeditation in the process of killing. If both competitors have joined a fight until death deliberately, then they both may perform an immoral deed. If one of them is dragged into the fight against her will and ends up killing the aggressor, then she may be exempt from moral judgment. Another question is how to perform the assessment of pleasure taken in the process of killing. Surely, we count as immoral the behavior of a serial killer who takes pleasure in torture of his victims. Still, we are concerned about the social problems related to a soldier with PTSD, even if he returned from a just war and was praised by the society for his service. The concept of "just war" and its implications are not obvious; it deserves a separate extensive analysis. For now, I am not too optimistic about the perspective of formulation of a consistent set of moral conceptual propositions framing a particular moral theory. The consistency of the set of moral axioms for any moral theory will depend on the referential machinery of our cognitive agency. It is a hard task. My goal is very humble. I need to explicate only one candidate proposition and show that epistemically this proposition behaves like a mathematical axiom. Also, to demand from a person to pose a consistent set of moral axioms, even for a narrow moral theory, is to demand to build a sort of strong homomorphism between mathematical and moral knowledge. Maybe it is possible, yet, it may take thousands of years. To finalize the problem of consistency of a set of moral conceptual truths for a particular moral theory, one more mathematical observation would be useful. The same or almost indistinguishable mathematical objects can be used in axioms for different fields of mathematics. The conceptual truths of respective sets of axioms are not afflicted by transition of objects from one set to another. Thus, a mathematician can contemplate the transition of the concept "line" from a Euclidean space to the non-Euclidean by imagining taking a straight line and simply bending it. A seeker of moral knowledge should be able to perform a similar transition of a moral concept. For example, one should be able to contemplate a broad concept of "objectification" containing manipulations, unjust labor, physical and psychological harm etc. This contemplation should allow a competent moral speaker to "bend" the concept of objectification in such a way that it fits, for example, a theory of "Killing". One may expect that providing an additional set of boundary conditions in order to restrict the application of the objectification would be enough. Yet, the consistent extraction of such conditions is a non-trivial task intertwined with the problem of applicability of moral concepts. It is also quite important to understand that consistency will depend on the logic used. Of course, we would like to embrace classic logic as an engine for our moral reasoning. Yet, it may happen that modal, or even fuzzy, logic will govern moral knowledge. It is unknown at the moment the precise form of desirable consistency of moral theories. Thus, a demand for moral conceptual propositions to be consistent in a similar manner as, for example, Euclidean axioms, is too early to ponder. Now we turn to the analysis of analogical behavior of moral concepts and corresponding mathematical objects.

First of all, we should remind ourselves that we try to present the ethical-mathematical analogy to proponents of mathematical Platonism and Logicism. Mathematical Intuitionist is also welcome to join us because, surprisingly, she could run into a similar problem of access to the knowledge of mathematical objects at the moment of transition from the concept of an ideal mathematical mind to the natural mind of a mathematician. As we saw in the analysis of philosophy of mathematics, we experience difficulties providing applicability and identity conditions even for the most basic mathematical objects, e.g. natural numbers. If one tries to define geometrical abstract objects such as a point or a line, she will be puzzled also. Dummett (1994) has proposed a way out through a concept of indefinitely extensible domain for mathematical objects, e.g. natural numbers. Now, let us note that Dummett's proposal is *analogous* to the Moorean Open Question argument about the non-natural essence of moral concepts. As long as one can indefinitely doubt that some natural property is indeed moral/immoral, one can doubt that number N is natural while grasping the totality of natural numbers. For example, remember that every natural number is finite but cardinality of natural numbers is not. Yet, this unclosed understanding of mathematical concepts does prevent us from having conceptual knowledge of mathematical propositions involving those concepts. Even if I am bewildered by the request to provide a clear definition of natural numbers, I am perfectly able to assert that “2” is a natural number and “2+2=4” is a true proposition. Let me reiterate this interesting situation. While a mathematician surely knows that Peano axioms of arithmetic are conceptual true propositions containing the concept of natural numbers, she is likely to experience an epistemic angst if asked whether she has grasped the totality of the concept. It seems that we may suppose an analogous situation with moral knowledge. One will have difficulty to define or grasp the totality of moral concepts; yet, she can be in a position to assert conceptual truth of a moral proposition containing two or more moral objects. Thus, this consideration gives us an important result. Our pure moral knowledge consists of propositions which are establishing relations between moral and other non-natural concepts. It is silly to ask whether the number "2" is true; we need a properly formed mathematical proposition establishing relations among several mathematical objects or concepts in order to think about the propositional truth value. Our very first question about moral knowledge of how to know that F is M, where F is a natural fact and M is a moral predicate, can turn out to be short of being a question about moral knowledge. One may object that we are just attempting to avoid the real problem of moral epistemology by proposing to see moral knowledge as a set of conceptual and inferential true propositions consisting of abstract, non-natural concepts. Indeed, one has all the rights to demand a survey of *applied moral theory*, if the search of pure moral knowledge turns out to be an abstract endeavor. Next chapter will be dedicated to practical applications of our account of moral knowledge. Well, it turns out that if we develop the ethical-mathematical analogy sufficiently deep, then we are in need of applied morality analogously with applied mathematics. Unfortunately, this area of applied morality is also likely to be afflicted by many epistemic troubles as we saw in the above analysis of applied mathematics.

Now we shall finalize our ethical-mathematical analogy through consideration of our candidate proposition for being conceptual: "objectification of one person by another person is immoral". It's important to note that I do not claim that I was able to come up with a polished enough example of the moral axiom. I can be even wrong that this particular proposition is conceptually true. It may happen that some other, more polished, conceptual truths entail this particular proposition. I also put aside an obvious similarity with Kantian moral thinking. However, our candidate seems to sufficiently exemplify the target analogy. First, the candidate proposition contains a concept of "person", or "personhood". I strongly reject a radical Constructivist account of this concept and the like. Though some features of "personhood" are obviously in correlation with one's social, psychological, and physical environment, there is a part of the concept that demands to be grasped akin to the totality of a mathematical concept's domain. One may say that by allowing the personhood to contain a mysterious property of totality, or rather holism, we infect the concept with some moral weight. Thus, our candidate proposition is a mere truism. I think that it can be labeled as a truism as long as grasping the totality of natural numbers can be labeled as learning to count. The definition of personhood is a victim of the Moorean Open Question argument. We will wonder indefinitely by picking up natural phenomena with the purpose to squeeze them into a definition; therefore, personhood is a non-natural concept. Yet, Lutz's (2015) objection fails to do justice to the proposition. Despite our inability to provide clear-cut definitions of terms "a person", "moral/immoral", and even "objectification", we are able to grasp them and assert a conceptual truth about their relationships.

Let us make a final check for disanalogy between mathematical objects and such concepts as "being a person". Mathematical objects are characterized by three properties: abstractness, existence, and independence. We already have shown that as long as one is not a proponent of radical social Constructivism, one can consider personhood as an abstract entity. The existence of the concept is underpinned by the existence of persons, for example, due to analogy with the previously mentioned concept of equator. The important disanalogy with mathematical objects arises when we consider independence. It is counterintuitive to suppose even weak independence: if there are no persons, then it is counterintuitive to think that there is a personhood, though a believer certainly can embrace the property of moral independence. Yet, as we did not want to exclaim that a search for mathematical basis is not mathematics but theology, we do not want to link moral epistemology with theology. Thus, the independence property is indeed a disanalogy. However, the important question is whether the disanalogy is epistemically crucial. I assert that disanalogy is rather metaphysical; it is not epistemological. Epistemologically, the needed grasp of personhood is indeed analogous to the grasp of geometrical concepts. Thus, we should consider the holism of such concepts. Dummett's (1994) indefinitely extensible domain highlights another facet of human cognitions. Even Dummett's totality entails holism in the act of grasping the concept. A somewhat mysterious ability to explicate an abstract geometrical concept of “line” from its instantiation drawn on sand is epistemically analogous to the ability to see a personhood by looking at a person.

Now we can conclude that a careful demonstration of ethical-mathematical analogy is possible. Despite some metaphysical disanalogies, we still can render this analogy as epistemically plausible. It was noted that the degree of its plausibility depends on a particular vein of philosophy of mathematics and selection of mathematical theories chosen as the source domain for the analogy. The next chapter is dedicated to the consideration of substantial objections to the analogy. In particular, there are possible objections to both the moral pluralism which follows from the ethical-mathematical analogy and to the practical applicability of the account of pure moral knowledge.

5. Discussion of the main argument and objections to it. Practical implications of Moral Intuitionism

In the previous chapter, we have proposed a scheme for ethical-mathematical analogy, in particular, its target domain. The target domain consists of several moral theories which, in turn, consist of sets of abstract conceptual, or non-inferential, truths serving as moral “axioms” and sets of inferential, often obvious, moral implications. The knowledge of the truth value and the content of those propositions requires a proper grasp of the totality and/or holistic nature of moral concepts. This sort of pure moral knowledge does not provide guidance what one should or shouldn't do. It provides an assessment of whether an action has or has not the property of “being moral/immoral”. The guidance for one's proper deeds shall be the responsibility of Applied morality. Applied morality uses abstract moral theories analogously to applied mathematics by building models of situations. The models can vary in their degree of satisfaction in practical applications. All of the above can be a good theoretical scheme. Yet, one can, and even must, object: this scheme seems to try to trick us by pushing all important moral problems down the slope, naming them just practical. I agree that without consideration of practical application of the proposed theoretical account, the ethical-mathematical analogy will be fruitless. Applied morality poses two major objections to our analogy. First is the problem of who can make proper modeling of morally laden situations and how they can do it. It is a problem of competent moral speakers. Second is the possible crucial disanalogy of mathematical pluralism and morally laden situations. Many of the following arguments were developed already (Korchevoi, 2023b; 2024). The refinement of those arguments dealing with the objections will be considered further.

5.1. Does moral abstract thinking render mathematical one?

As we saw, several mathematical philosophies take pure mathematical knowledge to be 'construed as independent of human mind and languages' (Clarke-Doane, 2019: 1). Thus, it is appealing to keep some degree of epistemic independence in Moral epistemology. We expect this independence principle to work contrary to radical Constructivism. It helps us to avoid a radical "psychological" interpretation of subject's cognitions for it is a label of subjectivity, or even bias. 'The principal reason that ethical knowledge invites analogies with mathematical knowledge is that both areas can appear to be a priori' (Clarke-Doane, 2019: 2). Yet, this approach must not ignore the mere existence of a person who performs some intellectual activity in order to obtain knowledge. Even in the most bizarre cases, epistemologists must say something about the protagonist of the case, e.g. that all things considered, she has normal perceptual and cognitive abilities. However, in the situation of mathematical-ethical analogy, we tend to expose a mathematician who does not discriminate between, for example, the complexity of the problems of the Set theory and those of Euclidean geometry, so an epistemologist can pick up any mathematical problem in order to find similarities or dissimilarities between mathematics and ethics. Clarke-Doane (2019) notes that providing a clear verbal formulation of the Axiom of Foundation of the Set theory is not a platitude; therefore, there is a disagreement among mathematicians. By analogy, it is fine to have one among ethicists. While I would like to have some additional support to demonstrate that ethical disagreement is not a problem, the degree of ethical disagreement seems to be much worse than the mathematical. It is hard to imagine two mathematicians who will disagree that, for example, Euclidean geometry is a consistent theory and that the set of its axioms is well formed. Thus, when one generalizes that 'there remain intractable disagreements over the axioms of all our mathematical theories' (Clarke-Doane, 2019: 2), she presents a false assumption. The true one is that there is a current disagreement over the axioms of *some* mathematical theories. It is up to mathematicians and, less likely, philosophers of mathematics to decide whether this disagreement will lessen in its width and degree. The ethical disagreement is much more propositionally widespread and widespread among people. Nevertheless, this observation does not ruin the prospect of mathematical-ethical analogy if we question that the ethical theories advance at the same pace as the mathematical. Does it make sense to suppose that mathematicians have gone too far and ethicists now have to catch up? After all, we try to use mathematical knowledge as a template for the ethical one, not otherwise. Mathematicians do not care whether their theories contradict the ethical ones.

Thus, the epistemic abilities of a thinking person in the analogy should not be erased form the picture, especially if we turn to practical applications of our moral theories. First of all, we should not mix up the practice of conceptual thinking and practical applications of a theory. Let us illustrate how mathematicians can and cannot think. What if we can find an intelligent agent who experiences difficulties with understanding Euclidean geometry? Here we meet 'Karl, who has no knowledge of substantial mathematics, say arithmetic or geometry' (Shapiro, 2009: 191). Luckily, Karl is quite proficient in logic, though Shapiro (2009) claims that only basic logic would suffice for the imaginary Karl's performance. So, Karl deduces all theorems from a set of Euclidean axioms and comes up with something very similar to a geometry textbook. Does he know Euclidean geometry? Do axioms look foundational to him? I doubt it, so does Shapiro (2009). What our imaginary protagonist lacks is practice in theoretical, conceptual thinking. Karl lacks basic knowledge of geometry which is learned by presenting examples of geometrical theorems and their proofs. Here comes an objection to the above premise that consistent mathematical theory enjoys better foundational status than any current ethical one: 'if it is immersion in the practice that makes the axioms obvious, then the axioms are not properly foundational' (Shapiro, 2009: 180). Note that "practice" here is not an empirical application of mathematical results but practice in theorizing and solving mathematical puzzles. By taking this objection for granted in an epistemological consideration, we may feel entitled to claim that a cognizing person should not have any experience at all even if it is an experience of thinking about a priori matters. It is a mistake. In particular, Shapiro (2009) seems to violate the correct application of the method of cases here by refusing imaginary Karl to have a proper referential ability. Let us be merciful and give our Karl a sheet of paper and a pen. We can ask him to make sense of what he deduced while he was sitting in his armchair. Can one be absolutely sure that Karl cannot have an insight and figure out that the terms in his logical formulas represent graphical objects such as points, lines, and planes? If by drawing triangles and lines Karl is able to find graphical proofs of several theorems, then, eventually, he may come to a proper grasp of geometrical concepts. Eventually, he may decide that there are self-evident propositions. The considered Shapiro's case which excludes practice, even such as theoretical mathematics, conceptual thinking, and puzzle-solving, is an attempt to ignore the reality of mathematics per se. Shapiro (2009) does agree that 'every well-formed sentence in a mathematical theory makes a fixed assertion about a fixed collection of objects and concepts'(177). He also states that a proposition is self-evident iff 'clearly grasping p is sufficient and compelling basis for recognition of p's truth' (Shapiro, 2009: 185). Self-evident propositions should not be grasped immediately and with obviousness as mathematical practice shows. Still, all the above concerns may bother a lot of philosophers of mathematics. For example, Hilbert's algebraic understanding of the role of axioms and Zermelo's version of self-evidence can be of great interest for philosophers of mathematics. Yet, it seems that even if something like Hilbert's metamathematics or Object Theory (Zalta, 2024) will allow to settle the speculations about philosophy of mathematics, it will unlikely change the consequences for ethics, i.e. widespread confidence that some mathematical truths are a priori truths and they bear epistemological status of being self-evident. Even one who critiques the concept of self-evidence of axioms in mathematics agrees that some theorems really are self-evident (Shapiro, 2009). Thus, we need to analyze whether an ethicist could or should have similar epistemic ability to grasp ethical theoretical concepts. By analogy, we will suppose that the situation of an ethicist is exactly the same.

Here we encounter another peculiar issue raised by Clarke-Doane (2019) - the unreliability of moral knowledge based on a mathematical «blueprint» due to the difficulties in practical decision-making procedures in ethically challenging situations. In other words, while reliability of mathematics can be explained through mathematical pluralism, the same strategy works poorly in ethics because 'there is something transparently unsatisfactory about ethical pluralism' (Clarke-Doane, 2019: 6). Zalta (2024) speculates that mathematical pluralism comes in three forms: consistent mathematical theories consisting of truths about their own domain of objects and relations, consistent and inconsistent mathematical theories, and different approaches to the philosophy of mathematics in general. Further, I will understand mathematical pluralism as the above first, most simple, formulation. Thus, the target question of our further speculations is the following: 'ethical theories are supposed to tell us what to do, and ethical pluralism leaves us clueless' (Clarke-Doane, 2019: 6). Does it though? Clarke-Doane (2019) thinks that a gap between, let us say, Deontological ethical theory and the Utilitarian one is unbridgeable in practice. Let us use a “toy” model of mathematics consisting of two theories. One is Euclidean geometry on a plane, the other - Hyperbolic geometry. They are consistent in themselves. They both contain some self-evident truths. If one is dissatisfied with their axioms, then she may dwell on the truths of the sum of angles of a Euclidean and hyperbolic triangles. At the very least, since one becomes familiar with the graphical proof that the sum of angles of a Euclidean triangle is 180 degrees, it is hard to undo the sense of one's certainty in it. The same applies to the negation of the analogous proposition related to hyperbolic triangles. Given the above mathematical pluralism, if a mathematician is asked what the sum of angles of a triangle is, she expects a clarification of what theoretical space is considered: Euclidean or Hyperbolic? Otherwise, the question does not make sense. Note, that only after conceptual assessment, we may consider the activity of an applied mathematician who is deciding which mathematical model to use in an apartment renovation or a satellite trajectory calculation.

As the case of a fat man on a footbridge persistently occurs in ethical discussions (Bedke, 2010), we will use it as a “moral-what-is-the-sum-of-angles” question. Thus, when an ethicist is asked whether one ought to push a fat man from a footbridge to save five, her answer should be: what theoretical, or conceptual, moral space is considered? Utilitarian or Deontological? The problem here is a devastating difficulty to discriminate between those spaces. Clarke-Doane (2019) asserts that the problem is with the analogy. I think that the problem is with the ethicists. What are the objects of ethical enquiries projected into the practical realm? It is a person and her behavior. Do we understand those objects and their underlying concepts? Do we have a proper grasp of their meaning? By a particular immersion in practice of theoretical thinking, a mathematician is likely to have a proper grasp of concepts of a point and a line. Yet, if asked to provide a clear definition of those geometrical objects, a mathematician encounters a difficult task. Ethical “points” and “lines” may be much more difficult to grasp. Let us briefly return to the above proposition that objectification of one person by another is immoral. This proposition involves a concept akin to personhood or humanness. We do not want to ignore the achievements of modern social sciences, including Constructivism. Those achievements allow us to expand the concept, to celebrate the dignity of the marginalized. However, the real goal of an ethicist is not a quest for the separation criteria; the real goal is to grasp and retain the universality of moral concepts, e.g. personhood. I do not want to dig deeper into terminological speculation here of what should be used as a key moral property of a personhood. It can be either totality, wholeness, or universality. As we saw, mathematical objects and moral concepts are metaphysically different. Thus, the moral property of, let us say, universality does not coincide with Dummett's indefinitely extensible domain of mathematical objects. Yet, the moral universality and/or totality can epistemically behave very similarly - they should be grasped. This grasp demands one's maturity. Nevertheless, human ethical hubris is like a virus. When Lutz (2015) asks whether we can doubt the ethical proficiency of Ancient Romans who did not care much about gladiators' games, my answer is that we can and must experience such doubts despite our respect for ancient philosophy. Shapiro (2009) ponders whether the grasping of moral concepts is enough to know conceptual moral truths. He thinks that it is not the case because ‘people who flout human rights can hardly be accused of not understanding the concept “men” (or “people”, or “human”)' (Shapiro, 2009: 205). What is this naive belief that everyone on this planet is morally knowledgeable yet, she deviates from morality because of some other non-moral obstacles? People who flout human rights must be accused of many things, including moral vices and misunderstanding of basic moral concepts.

Returning to the practical questions of the Trolley problem, let us suppose that the theoretical context of the problem involves a proper grasp of the concept of a person’s autonomy. All things considered, a person is entitled to have the right over her body and life. There are people who prefer to keep it this way. However, there are people who do not. By their own volition, they submit themselves to different modes of estimation of the value of their lives. For example, there are law enforcement and military forces. People were sworn to serve there; basically, they have committed to sacrifice their lives in order to save civilians who did not make such a commitment. Anyone can make a sacrifice. We praise her courage but we do not expect it. Yet, we expect much more from a person whose job it is to serve. We do not despise a utilitarian approach in a situation of war, for example. We do not question a commander's qualities who sacrificed a squadron to save an army. Thus, an ethicist should ask: who are those persons on the bridge? If they are civilians to the best of our knowledge, then none of them has obligations to push anyone. It is a deontological ethical space. Another situation occurs if one or both of them are, let's say, police officers. We would expect something from them. If they do nothing, we will likely ask them why. It is a utilitarian ethical space. Sure, structures of such social groups, their education and discipline tend to avoid the need of "pushing a fat man". It is a spiritually, socially, and psychologically traumatic event. Probably, because of that, a commander asks for volunteers to go on a deadly mission even though he could simply order someone to do it. So, we would love it if an officer "jumped" under the trolley by his own will. This proposal may seem unemotional but it fits the following statement about 'contrasting intuitions about doing-allowing cases; e.g. that it is impermissible to kill one to save five, while it is permissible to let one die to save five (Quinn, 1989, cited in Bedke, 2010: 1069).

It is useful to reiterate our general structure of argument which deals with the objections to the ethical-mathematical analogy. The set of objections pointing out to the different kinds of moral disagreement can be defeated by the supposition that across time and societies, the number of proficient moral speakers can be very small. If mathematics is a complex matter for one to be sure that she is good at it, then why does morality have to be different? The moral Intuitionism with the added ethical-mathematical analogy seems to provide not only a reasonable alternative for, let's say, evolutionary explanation of moral beliefs; it even gives us some arguments contrary to such explanation. For instance, one can find in the history of humankind a phenomenon of nurturing of groups of keenly violent people. The examples of such groups can vary from mythological berserkers to modern special forces. Of course, they were respected in some societies, especially in the past. I suppose that they loved to kill; they probably took pleasure in killing. Those groups also could play a significant role in keeping aggressive neighbors at bay. Yet, such groups were considered as somewhat socially alienated, something was not right with them. I suppose that a sort of moral disgust was connected with the possessive pleasure of killing exhibited by such groups. With the growth of moral consciousness, this moral intuition, i.e. wrongness of taking pleasure in killing, even when it is necessary, became clearer. Of course, one may respond that the clarity of the intuition can be a mere reaction to the increased safety of modern society. However, the dynamic of modern safety is a matter for discussion. Also, our concerns with the existence of obsessive killers, even though incorporated in our society, seem to have occurred earlier than we put surveillance cameras on every corner.

Also, the ethical-mathematical analogy which incorporates mathematical pluralism gives us a clue about how ethical pluralism can work. Still, the ethical space may be much more complex compared with the above case of the Trolley problem. Indeed, the complexity of applications of moral knowledge entails that a mere complaint about one's lack of moral proficiency will not do the trick. We need to provide some clues on how to discriminate between different moral conceptual spaces, especially in practice. Now we will turn to the problem of moral vagueness.

5.2. Moral vagueness

There is a strong objection to the mathematical analogy in Moral epistemology; it is a threat moral relativism poses in practical applications of moral pluralism if we employ mathematical pluralism as a natural development of the mathematical analogy. However, a closure shall be stated: we have moved to the consideration of practical application of ethical-mathematical analogy. Now, we are in the realm of “applied morality”. Thus, our problems now render a question of what one ought to do in this or that situation.

One more disclaimer is in order here. Below, I will use some mathematical theories to deal with vague predicates. Those theories do not develop our target domain for analogy. It can be the case that the problem of vagueness should be resolved through absolutely different kinds of theoretical thinking which has nothing in common with the mathematical one. My goal in this particular section of analysis is to paint an abundant tapestry of problems related to vagueness and, as it happens, to show that other philosophers do not hesitate to put “moral” variables into purely mathematical models.

The advocacy for moral pluralism has a big obstacle which should be addressed: the problem of “vague” predicates. Indeed, one can appeal to common sense by describing the following situation. Consider a person in everyday normal life. It is most likely that whichever ethical reasoning the person uses, she eventually builds some sort of utility function aiming to obtain as much good as possible while causing as little harm as possible. In our daily routine when everything is “business as usual”, we make compromises by considering our own interests as well as interests of our social surroundings; we exploit a utilitarian practical approach. Yet, there are moments when we feel that something is off. For example, one's neighbor is misbehaving by violating norms of the level of permissible noise. At first, it was nothing too big: just a loud voice. So, the person approaches her neighbor by starting a polite conversation resulting in a compromise: she does not file a complaint, the neighbor takes measures to comply with the norms. It goes like this for a while at a very steady pace. Not often, only once in a couple of months, the neighbor causes a noise, one tenth of a decibel louder each time. As a result of several conversations, the tension increases. Finally, the person calls the police. A policeman finds that a disgusting case of abuse and domestic violence takes place in the neighbor's household. When should the person stop to seek compromises and take actions? Was it too late already? It is an example of a “vague” moral predicate. In particular, “this behavior is permissible/impermissible”. As we go about our day, we tend to hold a position that such and such behavior is still permissible but it should be mitigated to better fit our or/and social interests. It is an area of utilitarian thinking. Yet, there is a moment when a switch takes place and we make the judgment that the behavior or its cumulative effect is impermissible anymore. In this moment, we leap into the realm of deontological ethics even if we may not consciously admit it. It is a form of practical moral pluralism. Whatever reasoning underlies our moral judgments, it seems that we are practically bound to moral pluralism. Without deontological borderline, we are doomed to moral relativism. However, forcing ourselves to jump into Kantian wagon whenever we encounter any social issue, we quickly end up an antisocial weirdo. The important question is how to make our moral agency better equipped to catch the moment of moral/immoral gestalt collapse.

The basic definition of vague predicates consists of the following conditions: a vague predicate has no sharp boundaries, at least known to us; the vague predicate contains borderline cases; the vague predicate is afflicted by Sorties reasoning (Dinis, 2017). Whether the last condition is necessary or sufficient is not clear. As one may argue, for some sort of vagueness, e.g. observational and/or phenomenological, the first two conditions plus the standard propositional logic will suffice to build a theoretical frame for the concept (Greenough, 2023).

Further, we shall cautiously proceed to assess what a moral vague predicate is. First, it seems reasonable to restrict the scope of our endeavor to the realm of human affairs. Second, there are predicates that reflect on a person's inner milieu, such as 'being happy' (Hawthorne, 2022: 215), being miserable, vicious, righteous, etc. Those predicates fall outside of the scope of Applied moral epistemology. Usually, a statement about someone's happiness bears information about that person's feelings but not a reflection of her objective inner moral quality. The same applies to labeling one as being good, bad, vicious, righteous etc. We are allowed to label some relations between moral concepts as being good/bad. Yet, it would be an area of theoretical, conceptual moral thinking. When the situation turns to the consideration of a particular person's life, such labels show rather the observer's psychological projections. It does not mean that one cannot build a Sorties series related to such predicates, but such a series should be studied rather in psychology, sociology, or theology. Moral epistemology strives for moral knowledge; it should not be concerned with the knowledge of psychological phenomena unless they contribute significantly to the context of moral judgment. I allow myself to draw a metaphorical line: if you feel guilty, see a therapist; if you are found guilty, go to prison. The difference resembles the situation when "one is tall" and "one looks tall", i.e. observational vs. phenomenological vague predicates (Greenough, 2023). Thus, it is not a surprise that Hawthorne (2022: 220) proceeds in his reasoning, without noting it, from one's feelings to the consideration of such thought experiments as fictional Darryl, a father who took his 6-months daughter to a park. So, for how long is it permissible for Darryl to divert his attention away from his daughter? The ethical question related to predicate “permissible” is legitimate, yet it is a question of one's action, behavior, and deed. We do not care how Darryl feels. We are concerned with what he does. Third, there is an issue of continuous predicates because 'the choice between continuity and anti-continuity for various moral scales is an intriguing one' (Hawthorne, 2022: 240). Since we've allowed ourselves to worry about the behavior of a person or a group only, we can realize that one's actions may be related either to continuous physical scales or to sets of physical phenomena. The example of the former is Darryl's case where the situation develops along the time scale. The supposition that time, length, volume, etc. are continuous phenomena serves our epistemological concerns properly. Of course, for the purpose of navigating our daily life, we have introduced different scales for the above entities; we make measurements. Yet, all those measurement tools are artificial. Despite us performing measurements using some sort of idealized templates, the reality of those physical scales is best described by continuous functions. Yet, we surely can build Sorties series using an arbitrary measurement scale. The question of Darryl's permissibility of attention diversion or proper distance from his daughter will rightly worry us. While Sorties “heap” is worrisome, the competition between Achilles and the tortoise is out of the scope of this article. Now, what can we say about moral predicates linked to sets of phenomena in the physical world, e.g. “being a doctor who saved five lives” or “being a serial killer who strangled six victims”? Those predicates look much simpler at first glance. They can be described by a discrete function and, therefore, they entail countability. Thus, an analogue of classic countable series of a “heap of wheat” can be built. It can be the case that continuous and discrete predicates demand different kinds of treatments for corresponding Sorties series. If such different treatments shall be found, the only problem will be that such schema lacks simplicity. Yet, who said that moral puzzles are easy to solve? The last proper note should be made here. In the above definition of continuous and discrete moral vague predicates, the properties of “being continuous and discrete” are related to physical realities which serve as variables for a moral “function” - the predicate. There could be other independent variables, e.g. contextual, epistemic, moral, etc., the characteristics of which are yet unknown to us. Also, the continuity/discontinuity of the moral outcome of the predicates (dependent variable) is not defined by the set of independent variables.

5.3. A brief analysis of possible treatments for Sorties paradox

There is a list of approaches to solving the Sorties paradox: epistemicism, contextualism, several kinds of supervaluationism and subvaluationism, many-valued logic, including fuzzy logic, and nonstandard analysis proposal (Dinis, 2017). The goal of this analysis is to present maximally practical algorithm to deal with moral Sorties series. I will not argue for the pro and contra of theoretical merits of the above approaches; I will critique them from a practical point of view.

Epistemicism, contextualism, supervaluationism, and subvaluationism are all afflicted by a particular shortcoming: it is very difficult to tailor those approaches into an algorithm for moral decision making. Epistemicism says that there is a sharp borderline in the series of propositions: "the n-th grain of wheat does/does not make a heap". Unfortunately, we do not know which one it is due to our weak ability to discriminate phenomena. Contextualism is even less certain saying that while all of the above propositions may be true separately, yet (n+1)-th proposition is false given that n previous propositions are uttered already. The question remains what the number n is. Supervaluationism (subvaluationism) proposes only coarse-grained solutions of the sort that n-th proposition in the series is true for sure and (n+k)-th proposition is false for sure (sometimes true and false for subvaluationism) with the borderline cases in between. Still, it is not entirely clear how to find those numbers n and k in the practical situations and why the search for those numbers does not create another Sorties paradox or at least the same degree of puzzlement. Thus, if Darryl asks a proponent of the above treatments for practical advice, he is likely to get the following answer: "You are ok diverting your attention from your daughter for a couple of seconds and you fail to fulfill your duty by doing it for 10 seconds. What is in between? Danger. Be cautious". It is better than nothing. Yet, Darryl might say that the adviser has made his life even more miserable because now he must figure out the rationality of two points of moral bifurcation instead of one: where the dangerous area starts and where it ends. The shortcoming of the above treatments is the flip side of their advantage. They all keep the standard propositional logic untouched. Further, we investigate the two remaining treatments which sacrifice, at least to some degree, the beauty and simplicity of propositional logic: many-valued logic and nonstandard analysis approaches.

Let us consider the nonstandard analysis approach. Nonstandard analysis utilizes a concept of infinitesimal and infinitely big numbers as entities which celebrate the same degree of rigor and reality as, for example, natural numbers. The theory was developed as an alternative for the Theory of limits and standard epsilon-delta procedure toward Differential and Integral Calculus. Dinis (2017) sees the nonstandard analysis as the most promising approach to solving the Sorties paradox. 'The heap arises when one has a nonstandard number $\omega \sim +\infty$ of grains of wheat' (Dinis, 2017: 14). Yet, there is a crucial objection toward this solution: 'a million grains of wheat should form a heap and yet that is clearly a standard number of grains' (Dinis, 2017: 15). In the area of Moral epistemology, the solution looks even more bizarre, e.g. we should not call a serial killer names because the total sum of his victims is a natural number. I think that the reason for the unsatisfactory application of the approach rests in the situation that Dinis (2017) does not discriminate continuous and discrete predicates. Let us restrict the consideration to a set of continuous predicates for now. Those predicates have, as a variable, some sort of continuous physical entity. The mathematical intuition says that in such case, if one wants to get an idea on how the function looks, she should make some calculus, take some derivatives, find extremes, etc. Thus, the scheme utilizing the notion of infinitesimals seems to be sound. For example, it solves immediately the issue of proportionality of moral response which concerns Hawthorne (2022). There will be no proportionality and it is not even an issue. A lot of social and physical situations look like complex systems, therefore, an infinitesimal change in one variable can cause a bifurcation in the system moving it into a new state. Note that even a description of a complex system resembles a Sorties series; complex system is a system with threshold behavior. A driver on the road diverts his attention from the road for more than (n+1) seconds and accidentally kills a pedestrian. The driver is guilty; it is very likely he will go to prison. Yet, his behavior is impermissible even if he did not kill anyone because he has substantially raised the probability of an accident. How do we know it? We calculated it, literally. Given the speed of a car, road conditions, time of reaction of an average person, etc. we know well enough the maximum of permissible time of attention diversion for one to drive safely, e.g. (n+ infinitesimal) seconds. Thus, one rightly gets a ticket for speaking on the phone while driving.

We may see that in the situation of moral vagueness involving continuous physical scales, the mathematical reasoning does not seem to be off of its limits as we must calculate a lot about our reality in order to navigate it. Yet, it is unclear why we should prefer the nonstandard calculus; the standard Differential and Integral Calculus will be sufficient in many cases of proper Sorties series. Thus, Dinis's (2017) intuition about the usage of the concept of infinitesimal numbers is insightful but his move to nonstandard analysis can overcomplicate the matter. Of course, some cases remain unresolved. They will probably remain unresolved for a long while. When is abortion permissible? This case involves not only the time scale but many undefined and unknown variables such as consciousness, the beginning of life, the context of conception, and even modal reasoning of one's life possibilities.

Further, I will consider the last approach to the Sorties series: many-valued logic. It will be applied to the discrete vague moral predicates such as "how many people should be discriminated against because of their race in order to call the society racist?" or "how many people can we remove form a heap of vicious population of Sodom and Gomorrah in order to spare them collective guilt and/ or responsibility?" (Genesis 18: 16-33). As we see, the issue has a long history. The major characteristic of the discrete vague moral predicates is that they are countable even if their limits can be infinite. Thus, they resemble the classic case of observational vague predicate of "how many grains of wheat make a heap". Also note that in applications of Moral epistemology, the majority of Sorties series is likely to display particular numbers for the first and the last propositions. For example, it is not enough to kill one person to call an event "a mass murder". It is obvious that killing a million can be called "a mass murder". Thus, the set of propositions of the "killing n people is a mass murder" kind is not only countable but finite in many, if not all, cases.

For the sake of brevity, I will omit the consideration of the applications of three-valued systems such as Kleene's and Lukasiewicz's. Those systems exhibit some shortcomings which standard propositional logic does not, e.g. absence of tautologies. Yet, they seem to possess less explanatory power for the purpose of practical application in comparison with fuzzy logic. Thus, further we consider a system with the truth degree set [0,1], in particular, a finite set of rational numbers within the interval. It was noted that fuzzy logic already has had interesting practical applications for a complex task solving, e.g. traffic problems (Dinis, 2017). The major objection to fuzzy logic application to Sorties paradox solutions is an arbitrary assignment of truth values to propositions, i.e. why the n-th proposition in the Sorties series has truth value 0. 971 rather than 0.972 or any other f(n) for this matter? Further I will try to overcome this problem by proposing a voting procedure in the field of moral judgments.

Voting is not a ridiculous artificial proposal in moral matters. Our society uses such explicit procedures, e.g. jury trial. Also, we have implicit surrogates, e.g. “public opinion” expressed by mass media. So, maybe we need to refine the procedure which we have actually used for millennia or, at least from time to time, since the trial of Socrates. Voting is a complex logical and mathematical problem studied by the social choice theory (Holliday, Kristoffersen, and Pacuit, 2024). Therefore, the further voting procedure does not claim to be the ideal solution for discrete moral vague predicates; it is rather an example of how to shed light on a particular approach. All the same, the Athenians can kill a genius. Still, we must know on whom the shame is.

Let us perform a thought experiment. Suppose that we wonder whether a particular political regime is abusing its power via «mass repressions» of its citizens. Suppose that we chose through an unknown, for the moment, procedure m competent moral speakers. This group of speakers has agreed that one repressed person does not comprise the term “mass repressions”. They have agreed that, for example, one million repressed people deserves to be called mass repressions. Now we ask this group of competent moral speakers to vote consecutively on the truth-value of the proposition “the event of n repressed persons is called mass repressions”. The voting goes for n from 1 to one million. All m voters count the proposition “the event of 1 repressed person is called mass repressions” as false. All m voters count the proposition “the event of one million repressed persons is called mass repressions” as true. Obviously, there is a number 1 < k < 1 million such that r voters see the proposition “the event of k repressed persons is called mass repressions” as true, where 1 < r < m. Of course, we expect a consistent voting, i.e. if a voter has decided that k-th proposition bears truth, then (k+1)-th proposition has the same value. Thus, we assign to k-th proposition the degree of truth which is a ratio r/m. In such a way, we obtain a clear procedure for truth values of corresponding propositions where the degree of truth is not an arbitrary number despite that the personal preferences of a particular voter could be quite subjective. For the sake of simplicity, we omit the consideration of how connectives should work with the fuzzy truth values of the above voting procedure. For example, one can consult the applications of proper mathematical and logical apparatus in the works where very similar proposals of voting in fuzzy logic were considered already (Gaines, 1976; Lawry, 1998). Thus, the above turn to voting is nothing new. I just wanted to illustrate that if we can function properly with a certain degree of contingency in the field of social choice, then fuzzy logic can be absolved from the sin of contingency in the field of moral vagueness in the same way.

It is also worth noting that the above example is rather a crude approximation because the term “repression” contains its own vagueness. Thus, some other procedure has to be set in order to define it.

Yet, one may object that the concern here isn't about contingency but rather that of the subjectivity of the voters. It is true. Nevertheless, until we call for a moral authority of an omnipotent being in every possible situation, we are bound to lean on the practical expertise of real humans. Now we shall speculate on the matter of who those competent moral speakers are. As I see it, this is the most important question not only for the above case but for the whole project of Moral epistemology.

5.4. A competent moral speaker: is it easy to find one?

I do not dare to think that I am able to answer this question as the whole history of humanity seems to be an unsuccessful attempt to do so. Since human societies have developed beyond their mere survival, they introduced religions and religious speakers of many kinds. Those speakers have served as moral speakers, at least in some cases. Unfortunately, since the agency of priesthood was robbed of its moral authority due to industrial development, probably rightly so, we have an absence of social institutions that claim moral authority. Of course, we are able to point out a lot of substantial shortcomings of religious organizations and their corresponding values. Yet, what is important here is that there were organizations who announced that their goal was to seek moral excellence. If today one wants to learn morality, where should she go? Psychology took the place of the priesthood, soothing feelings while having an imperative of avoiding moral judgements. Psychology teaches us that every feeling has its own reality and, therefore, the right to be considered seriously. While this position helps to build a safe communication between a psychologist and her client, it almost does not leave space for appeal to universal concepts. Modern post-modernist realm of social media with a bunch of celebrities and influencers advising on everything fits the role even more poorly than a session with a good existential therapist. Yet, unable to provide the needed answer, we can formulate at least some necessary conditions for a competent moral speaker from an epistemological perspective.

We already noted that the situation with discrete moral vague predicates and countable, likely finite, Sorties series resembles classic vagueness with observational predicates, i.e. one sees that something is a heap of wheat. For observational predicates, we have a necessary N-condition in order to be a competent observer: the observer functions normally by having proper cognitive and physiological abilities; no external barriers preclude an observer's ability to observe an object (Greenough, 2023). Thus, we can try to formulate analogous internal and external conditions for a moral speaker.

First, we consider the external condition: it is necessary for a moral speaker not to have a conflict of interests and personal negative consequences as a result of the speaker's utterances. For the above example of voting related to the mass repressions “heap”, it means that the speaker does not have hostages taken by oppressive power and lives in relative safety. Yet, the moral external condition has to take into account, let us say, moral distance of a speaker from a particular phenomenon. We expect from the speaker to be familiar with the context and have proper experience. Thus, our candidate to fulfill moral assessment cannot be chosen from a social group or strata that consists of people who cannot find a country with abusive regime on a map.

Second, we turn to the internal speaker's moral abilities which is a much more puzzling matter. Just as we expect from an observer of a heap of wheat to have good eye sight, we expect a moral speaker to be a normally socializing person who behaves more or less respectfully. So, the necessary condition isn't a demonstration of a virtue, but rather an absence of an extreme vice, e.g. being a sociopath. Unfortunately, we are likely to stumble in the next step. We can describe a normal cognitive function for an observer of a heap of wheat. At the very least, the observer must understand what wheat and a grain of wheat are. What should a moral speaker understand while judging a “heap” of repressions? Trying to catch more viable subtlety, who is better equipped to grasp the morality of “killing”: a soldier or a civilian? Those questions do not have obvious answers. Yet, they give us a hint for the following argument.

Let’s remember that we are speculating about practical implications of our ethical-mathematical analogy here. Thus, it is not a solely theoretical conceptual conversation. We rightly can appeal to a way of thinking exhibited by mathematicians by proposing for ethicists to do similarly. Let me pose an observation. There is a few mathematicians who would claim that they are equally proficient in all mathematical theories. For example, I have a solid educational background in the Theory of probability. After a decent consideration, I would probably say that I can grasp the depth of problematics posed by counterintuitive results of some probabilistic phenomena. Yet, I would never claim that I can understand Topology well enough to grasp its modern achievements, in particular Perelman's proof of Poincare hypothesis. Thus, it can be the case that there are no universally proficient moral judges; the competence in one area of moral knowledge does not entail the same epistemological abilities in others.

It was noted that human moral competency is likely a pivotal issue for the project of proper applications of Moral epistemology. However, it is important to state that to supply moral knowledge is not a handy task for an “armchair” epistemologist. Epistemologists should rather ponder on how and when an observer sees a heap but try not to substitute the observer's decision with their own. Similarly, moral epistemologists should not preach morality but propose an analysis of what moral knowledge is. Thus, I leave the issue of the formation of social institutions producing morally competent speakers to other areas of humanitarian science.

However, in the flow of the above arguments, one may find a contradiction. Our society seems to have become less violent (Roser, 2013); still, this benevolent trend takes place in the society where moral judgements are often implicit or inconsistent. One can even be tempted to fall for moral skepticism seeing this process as a result of technological development, i.e. growing level of satisfaction of basic needs entails a decline in average level of aggression. Well, one may say, there is no morality but economic, social, and psychological implications of technological development in play. The above correlation between technology and social milieu can be the case. However, the correlation does not entail absence of moral knowledge. We may interpret the situation that our civilization has only been lucky so far: while having developed a lot of dangerous technological tools, it has not developed any explicit social respectful structures for moral assessment. What usually happens when technology takes a turn undesirable for us? Several voices declare: I told you so! Often those voices are linked with the failed technical expertise. Less often, we hear that this or that technological turn was immoral in the first place.

5.5. Supervenience objection

The final objection which must be considered is rather a metaphysical one: the supervenience objection. Yet, the objection can be properly addressed only after the above analysis of practical implications of Moral Intuitionism. Preliminary note is needed before we attempt to tackle the objection. The above analysis of moral vagueness has used different mathematical theories and logics. We noted infinitesimals and limits, fuzzy logic and mathematical choice theory. It is important to understand that those mathematical tools are not the only possible way to resolve the issue of moral vagueness. It may be the case that some other epistemological theory will prove its usefulness and we will not need to apply complex mathematics to highlight morally vague situations. It is rather amusing how often mathematics pops up in areas of reasoning seemingly far from it. Our ethical-mathematical analogy pursues a more modest goal. The underlying idea was that if we can see conceptual similarities between mathematical objects and moral concepts, then we can allow moral pluralism by analogy with mathematical pluralism. Yet, if moral pluralism takes place, then the problem of vague predicates should be addressed.

Now we turn to the supervenience objection. Our analogy obviously gives an account of moral knowledge as non-natural knowledge. The supervenience objection asserts that non-naturalism is not able to explain a supervenience intuition, i.e. moral properties supervene on natural properties. The objection poses the following intuitive argument: there cannot be a situation where moral assessments are different while the natural facts are the same. Indeed, it would be odd to suppose that while we retain all our knowledge about the Nazi regime, we can allow the existence of different moral judgments about it. In this way, moral properties supervene on natural properties and, therefore, moral properties are reducible to the natural. Moreover, it seems that we also have an analytical reducibility because we can form the implications of the following sort: a change in natural property can entail a change in moral property but not vice versa.

Our response to the supervenience objection will be counterintuitive. Yet we saw that Moral epistemology is likely to be a complex matter. We encountered moral vagueness and it seems to be our foe; we would like to clear the vagueness completely. However, it may turn out to be impossible. There will be situations where moral judgment cannot be performed using a clear output of vague moral predicate. In such circumstances, different moral theories can be applied and different moral assessments can be made. They probably will be unsatisfactory to a certain degree, yet we often find ourselves in the situation of the choice between the bad and the worse. For example, let's consider abortion. One position may be that all life, even its mere possibility, is sacred. Another position can defend, all things considered, a bigger value of the existing life in comparison to possible life. Thus, in the situation where the process of giving birth is a threat to a potential mother's life, any decision can bear an ambiguous moral assessment. Thus, we have a situation where natural facts are set, yet moral judgements can vary. I omit here many possible ramifications of the case, such as questions regarding when life begins, unwanted conception as a result of rape, etc. Again, we encounter a somewhat fruitlessness of our account of moral knowledge. However, the difficulty of applying the knowledge does not inform us about its falsity. When one is standing on a plot of land, she is right to apply Euclidean geometry. Yet, if one observes our planet from space, she immediately grasps that her observations are better covered by non-Euclidean geometry. Somewhere between those two situations, we may find a location where both geometries are applicable; it is likely that both geometries will produce errors if we use them as mathematical models. Our response to supervenience objection goes in the spirit of Shafer-Landau's (2003) views that natural properties constitute an instantiation of a moral property. Shafer-Landau argumentation supports the metaphysical premise of independence of moral concepts which I discuss in the final chapter. However, I deliberately avoid the metaphysical facet of the problem here. Sometimes it is better to practice something than to feel stuck due to the premonition that the practice is not going to be ideal.

6. Conclusion

In conclusion, I would allow myself to highlight some results of the above analysis that might bear novelty. First, I adopted a position of broad mathematical pluralism in the philosophy of mathematics. To settle the disagreement among a wide range of philosophies of mathematics was not my goal. However, it was demonstrated that the proponents of weak Platonism, Logicism, and mathematical Intuitionism may probably have an epistemological attitude and beliefs consistent with an ethical-mathematical analogy.

What common ground can we find in these philosophies of mathematics? I propose to label them as philosophies supporting the thesis of the discovery of mathematics. Even though one asserts that mathematical Intuitionism is explicitly about constructive mathematics, I would argue that mathematical Intuitionism entails a deep metaphysical commitment to the concept of an ideal mind. Thus, when contemplating the cognitive activity of a human - a mathematician - it is reasonable to ask: if a mathematician re-invents the results of an ideal mind, then how is it different from the discovery of mathematics? Even mathematical Intuitionists should somehow propositionally express their adherence to the acts of intuition of an ideal mind. In this way, all proponents of the above philosophical approaches are committed to a presumably small set of true, abstract, conceptual propositions. The mathematical knowledge of those propositions is obtained non-inferentially. It should neither be immediate nor obvious. This knowledge may bear simplicity and beauty. The set of such propositions can serve as a source domain for the analogy between mathematical knowledge and moral knowledge.

The philosophical merits of the analogical argument were considered. It was stated that the result of a good analogy is only a plausible hypothesis. This seemingly weak result is sufficient for the aim of this dissertation because it strives to weaken the grip of the other hypothesis that all moral properties can be explained in terms of natural properties.

The proponents of many forms of mathematical Constructivism will not be satisfied with our analogy because their views do not allow any knowledge to transcend the realm of concrete objects and human cognitions. However, they must admit that if every moral property is natural property, then there can be situations where any malice is permissible. Moreover, such a position entails not only the permissibility of malice but also prevents us from uttering moral judgements. Thus, they must abandon the realm of morality and dwell in the purely natural realm: if the immorality of slavery was a product of social and economic development, then it would be honest not to use the term "immorality" at all. Still, there is space for future research because mathematical Constructivism is rather a spectrum of philosophical views. Consequently, it would be useful to specify where on this spectrum the concept of an ideal mind meets the cognitive activity of a human. The scrutiny of the implications of mathematical Constructivism for the ethics has to be conducted.

By carefully inspecting our source and target domains in the analogical argument, I attempted to highlight both similarities and dissimilarities between mathematical objects and moral concepts. It was noted that properties of abstractness and existence are similar in both the source and the target domains. However, an important dissimilarity was also found: the property of independence. While the independence of mathematical knowledge for proponents of mathematical Platonism, Logicism, and Intuitionism is intuitively consistent with those philosophical positions, the independence of moral concepts is counterintuitive. I tried to circumvent this obstacle arguing that despite this metaphysical distinction, objects in both domains of analogy epistemically behave similarly. They should be grasped by intuition and practice in conceptual thinking performed by a competent expert. The grasp entails some understanding of totality, and/or wholeness, and/or holism of concepts in both source and target domains of analogy. Thus, our result is as follows: if one is convinced that mathematics is discovered, s/he should also accept the existence of knowledgeable non-inferential, non-natural true moral propositions.

Yet, one can be disappointed with such a tricky epistemological turn and view the above dissimilarity as critical. If this dissimilarity is critical, then the analogical argument does not withstand the objection. Thus, I feel the need to state here that in order to uphold the analogical argument, we need not only a clear stand on the matter of the philosophy of mathematics but also on one more premise. This premise is metaphysical and it is the riskiest one. It is quite easy to suppose that if there were no intelligent agents in the universe, there would still be mathematics, at least in the form of laws governing the universe. For example, if all humans disappeared, a certain “number” of planets in the solar system would not disappear. The absence of intelligent agents does not imply a reign of chaos in the universe. We shall propose the premise that postulates a similar state of affairs in the realm of moral knowledge. We can call it the weak independence of moral concepts: if there were no intelligent agents, there would be moral concepts in the universe. I do not want to formulate here a final version of such a premise. As an illustration, I would propose the idea that since the universe is a place not hostile for life to occur, then the universe bears moral value per se.

Obviously, such a premise is rather quasi-religious in the above Prichard's (2016) sense and can play the role of Prichard's “über” hinge commitment in Moral epistemology. Nevertheless, it is crucial to understand that the account of moral knowledge obtained through the ethical-mathematical analogy does not presuppose the above-mentioned Prichard's interpretation of basic conceptual truths as exactly commitments. I agree with Zhang (2021) that a simple proposition and the Prichard's commitment are phenomenologically similar. Pritchard should insert some foundational meaning into those basic propositions in order to overcome obstacles with the Closure Principle.

The ethical-mathematical analogy, if understood as an analogy between realms of abstract objects, does not suffer from the Closure Principle trap. The CP problem occurs at the level of Applied morality where models of moral conceptual space use real situations as instantiations. The ethical-mathematical analogy informs us about the plausibility of the existence of non-inferential, non-natural, true moral propositions. Whether those propositions are commitments or just basic beliefs is a matter of deeper metaphysical research. By the same token, we do not assert that mathematical axioms should necessarily play the role of mathematical über commitments. They will play the role of basic blocks for mathematical theories irrespectively from philosophical re-naming. As we saw above, Prichard's (2017) solution began to shine in the cases where mathematical knowledge was considered as supervening in the realm of concrete objects. If we allow mathematics to be independent of concrete objects, then appealing to the need for an additional foundational status of our beliefs is excessive.

By considering the above metaphysical premise of weak independence of moral concepts, I am not trying to give a theological vector to Moral epistemology, though such a vector does not contradict my analysis and certainly can be a vein for future research. The additional premise raises the plausibility of the ethical-mathematical analogy. Yet, without such a premise, the analogy is still epistemically plausible. However, the idea that the concepts of “good” and “bad” transcend our mere existence as a species is not new; it is deeply rooted in our philosophy from the beginning.

It is worth noting that several well-known objections toward Moral Intuitionism were considered, particularly those related to moral disagreement and moral pluralism. Though, in my view, those objections do not pose a significant threat to the ethical-mathematical analogy, they give directions for future research, which I will list here. There is a question of which logic better suits the moral reasoning. We saw that different moral problems, especially practical ones, may demand an application of different logics: from classic logic to fuzzy logic. Also, it seems important to scrutinize the behavior of the negation operator in moral reasoning and probable asymmetry in our ability to formulate moral propositions: we do it easier by uttering a set of taboos in comparison with asserting rules for being righteous. The final issue demanding future research is the problem of competent moral speakers. It is not only a problem of providing conditions for their assessment; most importantly it is a problem of nurturing such experts. As I argued, it can be the case that Abraham's negotiation with God in the vicinity of Sodom and Gomorrah will be needed again. Thus, it would be good to have skilled negotiators at such a moment.

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